Inhomogeneous pressure models mimicking ΛCDM universe

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References

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Content:

- 1. Universe symmetries. Acceleration as back-reaction of inhomogeneities.
- 2. Complementary models of the spherically symmetric Universe.
- 3. Redshift drift in inhomogeneous pressure models.
- 4. Off-center observers and supernovae.
- 5. More tests in progress - central observer (BAO, CMB shift parameter).
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1. Universe symmetries. Acceleration as back-reaction of inhomogeneities.

In the context of dark energy problem (Λ being 120 orders of magnitude too large) there has been more interest in the non-friedmannian models of the universe which could explain the acceleration only due to inhomogeneity (initially E. Kolb). One of the strogest claims was that

we are living in a spherically symmetric void of density described by the Lemaître-Tolman-Bondi dust spheres model

e.g. Uzan, Clarkson, Ellis (PRL, **100**, 191303 (2008))
Caldwell and A. Stebbins (PRL, **100**, 191302 (2008))
Clarkson et al. (PRL, **101**, 011301 (2008))
and many others

Is there a center of the Universe?

- Simplest inhomogeneous models are **spherically symmetric** and violate the **Copernican Principle** which says that we do not live at the center of the Universe.
- However observations (including CMB) have been made just **from one point** in the Universe and extend only onto the one (and unique) past light cone and proved isotropy only.
- It seems that homogeneity **needs a check**.
- Suppose we live in an inhomogeneous model of the Universe with the **same (small) number of parameters** as a homogeneous dark energy model and they both fit observations very well.
- Could we **differentiate** between these models?

New "paradigm" of inhomogeneity - LTB void.

- There are **two complementary spherically symmetric models** of the universe.
- **These are: the inhomogeneous density (dust shells) Lemaître-Tolman-Bondi (LTB) models and inhomogeneous pressure (gradient of pressure shells) Stephani models.**
- Can they mimic homogeneous ΛCDM dark energy models?
- Additionally, there are lots inhomogeneous models (Goode (1986), Szafron (1977), Szekeres (1975), Wainwright-Goode (1980), Ruiz-Senovilla (1992) etc.) to investigate as candidates for dark energy. See e.g. A. Krasiński, K. Bolejko, M.-N. Célérier and others.

2. Complementary models of the spherically symmetric Universe

In order to make a **complementary analysis** with LTB models the following table proves useful:

	pressure	density
FRW	$p = p(t)$	$\varrho = \varrho(t)$
LTB	$p = 0 \ (p(t))$	$\varrho = \varrho(t, r)$ - nonuniform
Stephani	$p = p(t, r)$ - nonuniform	$\varrho = \varrho(t)$

SS Lemaître-Tolman-Bondi Universe

– is the only spherically symmetric solution of Einstein equations for **pressureless matter** ($T^{ab} = \varrho u^a u^b$) and no cosmological term (G. Lemaître, Ann. Soc. Sci. Brux. **A 53**, 51 (1933); R.C. Tolman, Proc. Natl. Acad. Sci., **20**, 169 (1934); H. Bondi MNRAS **107**, 410 (1947))

$$ds^2 = -dt^2 + \frac{R'^2}{1-K} dr^2 + R^2 (d\theta^2 + sin^2\theta d\phi^2) \quad , \tag{1}$$

where

$$R = R(t,r); \qquad R' = \partial R / \partial r; \qquad K = K(r) \quad . \tag{2}$$

The Einstein equations reduce to

$$\dot{R}^2 = \frac{2M(r)}{R} - K(r); \qquad 2M' = \kappa \varrho R^2 R' \quad , \tag{3}$$

and are solved by

SS Lemaître-Tolman-Bondi Universe

$$R(r,\eta) = \frac{M(r)}{K(r)}\Phi'(\eta); \qquad t(r,\eta) = T_0(r) + \frac{M(r)}{K^{3/2}(r)}\phi'(\eta) \ , \tag{4}$$

where for $K(r) < 0$ (hyperbolic), $K(r) = 0$ (parabolic), and $K(r) > 0$ (elliptic) appropriately ($K(r)$ is a spatially dependent "curvature index") we have

$$\Phi(\eta) = (\sinh\eta - \eta; \eta^3/6; \eta - \sin\eta) \ . \tag{5}$$

Regularity conditions:

- existence of a regular **center of symmetry** $r = 0$ – implies $R(t,0) = \dot{R}(t,0) = 0$ and $M(0) = M'(0) = K(0) = K'(0) = 0$ and $R' \to 1$.

- hypersurfaces of constant time are **orthogonal** to 4-velocity and are of topology S^3 – implies the existence of a second center of symmetry $r = r_c$ (with some 'turning value' $0 < r_{tv} < r_c$)

- a 'shell-crossing' singularity should be **avoided** – implies $R'(t,r) \neq 0$ except at turning values (though it is a weak singularity - no geodesic incompleteness)

SS Stephani Universe

– is the only spherically symmetric solution of Einstein equations for perfect-fluid energy-momentum tensor ($T^{ab} = (\varrho + p)u^a u^b + pg^{ab}$) which is **conformally flat** and **embeddable** in a 5-dimensional flat space (H. Stephani Commun. Math. Phys. **4**, 167 (1967); A. Krasiński, GRG **15**, 673 (1983)). After introducing a Friedmann-like time coordinate (cf. later) we have

$$ds^2 = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] , \tag{6}$$

where

$$V(t,r) = 1 + \frac{1}{4}k(t)r^2 , \tag{7}$$

and $(\ldots)^{\cdot} \equiv \partial/\partial t$. The function $a(t)$ plays the role of a **generalized scale factor**, $k(t)$ has the meaning of **a time-dependent ”curvature index”**, and r is the radial coordinate.

SS Stephani universes

The energy density and pressure are given by

$$\varrho(t) = 3\left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k(t)}{a^2(t)}\right], \tag{8}$$

$$p(t,r) = \varrho(t)\left\{-1 + \frac{1}{3}\frac{\dot{\varrho}(t)}{\varrho(t)}\frac{\left[\frac{V(t,r)}{a(t)}\right]^{\cdot}}{\left[\frac{V(t,r)}{a(t)}\right]}\right\} \equiv w_{eff}(t,r)\varrho(t), \tag{9}$$

and generalize the standard Einstein-Friedmann relations

$$\varrho(t) = 3\left(\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k}{a^2(t)}\right), \tag{10}$$

$$p(t) = -\left(2\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} + \frac{k}{a^2(t)}\right) \tag{11}$$

to inhomogeneous models.

SS Stephani universes

Kinematic characteristic of the model:

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} - \dot{u}_a u_b \ , \qquad \dot{u} \equiv (\dot{u}_a \dot{u}^a)^{\frac{1}{2}} \ . \tag{12}$$

where $\dot{u}$ is the acceleration scalar and the **acceleration vector**

$$\dot{u}_r = \frac{\left\{ \frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right] \right\}_{,r}}{\frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right]} \tag{13}$$

while the expansion scalar is the same as in FRW model, i.e.,

$$\Theta = 3\frac{\dot{a}}{a} \ . \tag{14}$$

Compare: LTB has non-zero expansion and shear.

Inhomogeneous pressure models - null geodesics

The four-velocity and the acceleration are

$$u_\tau = -\, c\frac{1}{V}\,, \qquad \dot{u}_r = -\, c\frac{V_{,r}}{V}\,. \tag{15}$$

The components of the vector tangent to zero geodesic are

$$k^\tau = \frac{V^2}{a}\,, \quad k^r = \pm\frac{V^2}{a^2}\sqrt{1 - \frac{h^2}{r^2}}\,, \quad k^\theta = 0\,, \quad k^\varphi = h\frac{V^2}{a^2 r^2}\,, \tag{16}$$

where h = const., and the plus sign in applies to a ray moving away from the centre, while the minus sign applies to a ray moving towards the centre. The **acceleration scalar** is

$$\dot{u} \equiv (\dot{u}_\mu \dot{u}^\mu)^{\frac{1}{2}} = \frac{V_{,r}}{a} = \frac{1}{2}\frac{k(t)}{a(t)} r \tag{17}$$

The **farther away** from the center at $r = 0$, the **larger the acceleration**.

Inhomogeneous pressure models - topology & singularities

- Global topology still $S^3 \times R$. The models are just **specific deformations of the de Sitter** hyperboloid near the "neck circle", **but with local topology of the constant time hypersurfaces (index $k(t)$) changing in time.**
- Usually we cut hyperboloid by either $k = 1$ (S^3 topology), $k = 0$ (R^3) or $k = -1$ (H^3) – here we have "3-in-1" – the Universe may either **"open up" or "close down"**.
- standard **Big-Bang** singularities $a \to 0$, $\varrho \to \infty$, $p \to \infty$ are possible (FRW limit).
- **Finite Density (FD)** singularities of pressure appear at some particular value of a radial coordinate r – in standard FRW cosmology there exist exotic (sudden future) singularities of pressure (SFS) with finite scale factor and energy density – they differ (MPD 2005).
- There is **no global equation of state** - it changes from shell to shell and on the hypersurfaces $t = \text{const}$.

Exact classes of inhomogeneous pressure models ...

... have been found (MPD, 1993). For the so-called **Model II** one has

$$\left(\frac{k}{a}\right)^{\cdot} = 0 \quad \text{i.e.} \quad k(\tau) = -\beta a(\tau) \tag{18}$$

with the unit $[\beta] = Mpc^{-1}$.

A subcase of model II (from now on **IIA**) was proposed by Stelmach and Jakacka (2001)) – it assumes that the **standard barotropic** equation of state

$$\frac{p(\tau)}{c^2} = w\varrho(\tau) \tag{19}$$

at the center of symmetry and **no exact form** of the scale factor. This assumption gives that

$$\frac{8\pi G}{3c^2}\varrho(\tau) = C^2(\tau) = \frac{A^2}{a^{3(w+1)}(\tau)} \quad (A = \text{const.}) \tag{20}$$

and allows to write a generalized Friedmann equation as

Exact inhomogeneous pressure models

and

$$\frac{p(\tau)}{c^2} = \left[w + \frac{\beta}{4}(w+1)a(\tau)r^2 \right] \varrho(\tau) = w_{eff}\,\varrho(\tau) \quad . \tag{22}$$

Similarly as in the Friedmann model, we can define critical density as

$$\varrho_{cr}(\tau) = \frac{3c^2}{8\pi G} \left(\frac{a_{,\tau}}{a(\tau)} \right)^2 \tag{23}$$

and the density parameter $\Omega(\tau) = \varrho(\tau)/\varrho_{cr}(\tau)$ which after taking $\tau = \tau_0$ gives

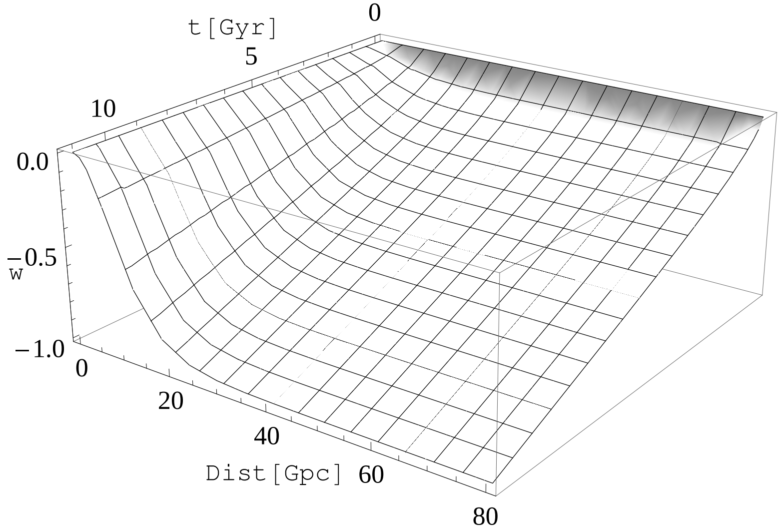
$$1 = \frac{A^2}{H_0^2 a^{3(w+1)}(\tau_0)} - \frac{\beta c^2}{H_0^2 a_0} \equiv \Omega_0 + \Omega_{inh} \quad , \tag{24}$$

and so

$$\beta = \frac{a_0 H_0^2}{c^2} \left(\Omega_0 - 1 \right) \quad . \tag{25}$$

Model I - effective barotropic index

Figure 1:



The effective barotropic index w_{eff} is getting more and more negative simulating dark energy for large distances form the center (at $r = 0$) and far from the big-bang singularity (at $t = 0$).

Exact inhomogeneous pressure models

In Model **IIB** the scale factor is of the **dust-like type**

$$a(t) = \sigma t^{2/3},\ k(t) = -\alpha\sigma a(t),\ \ , \tag{26}$$

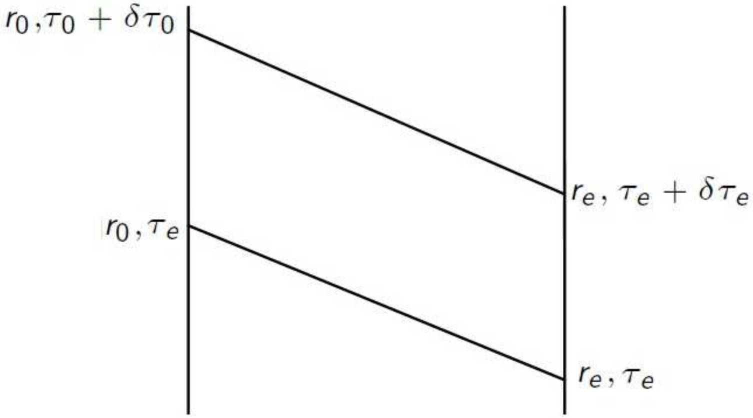
($[\alpha] = (s/km)^{2/3} Mpc^{-4/3}$, $[\sigma] = (km/s)^{2/3} Mpc^{1/3}$, $[t] = sMpc/km$) but the equation of state at the center of symmetry is no longer barotropic:

$$\rho = p\left(\frac{32\pi^2 G^2}{3\alpha^3 c^8} p^2 - \frac{3}{2}\right)\ \ . \tag{27}$$

In the limit of the **inhomogeneity parameter** $\alpha \to 0$ one obtains the Friedmann universe. FD singularity of pressure is at $r \to \infty$.

3. Redshift drift test for inhomogeneous models.

Redshift drift (Sandage 1962) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.



There is a relation between the times of emission of light by the source τ_e and $\tau_e + \delta\tau_e$ and times of their observation at τ_o and $\tau_o + \delta\tau_o$:

$$\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e+\delta\tau_e}^{\tau_o+\delta\tau_o} \frac{d\tau}{a(\tau)} , \tag{28}$$

which for small $\delta\tau_e$ and $\delta\tau_o$ reads as $\frac{\delta\tau_e}{a(\tau_e)} = \frac{\delta\tau_o}{a(\tau_o)}$.

Redshift drift in inhomogeneous pressure models.

For small $\delta\tau_e$ and $\delta\tau_o$ we expand in Taylor series

$$\begin{aligned}(u_a k^a)_o &= (u_a k^a)(r_0, \tau_0 + \delta\tau_0) = (u_a k^a)(r_0, \tau_0) + \left[\frac{\partial(u_a k^a)}{\partial\tau}\right]_{(r_0,\tau_0)} \delta\tau_0 \\ (u_a k^a)_e &= (u_a k^a)(r_e, \tau_e + \delta\tau_e) = (u_a k^a)(r_e, \tau_e) + \left[\frac{\partial(u_a k^a)}{\partial\tau}\right]_{(r_e,\tau_e)} \delta\tau_e \ ,\end{aligned}$$

where for inhomogeneous pressure models the readshift reads as

$$1 + z = \frac{(u_a k^a)_e}{(u_a k^a)_O} = \frac{\frac{V(t_e,r_e)}{R(t_e)}}{\frac{V(t_0,r_0)}{R(t_0)}} \qquad (29)$$

From the definition of the redshift drift by Sandage (1962):

$$\delta z = z_e - z_0 = \frac{(u_a k^a)(r_e, \tau_e + \delta\tau_e)}{(u_a k^a)(r_0, \tau_0 + \delta\tau_0)} - \frac{(u_a k^a)(r_e, \tau_e)}{(u_a k^a)(r_0, \tau_0)} \ , \qquad (30)$$

Redshift drift in inhomogeneous pressure models.

For general SS Stephani metric we obtain

$$\frac{\partial}{\partial \tau}\left(u_a k^a\right) = -\left(\frac{1}{a}\right)^{\cdot} - \frac{1}{4}\left(\frac{k}{a}\right)^{\cdot} r^2 \, , \tag{31}$$

and

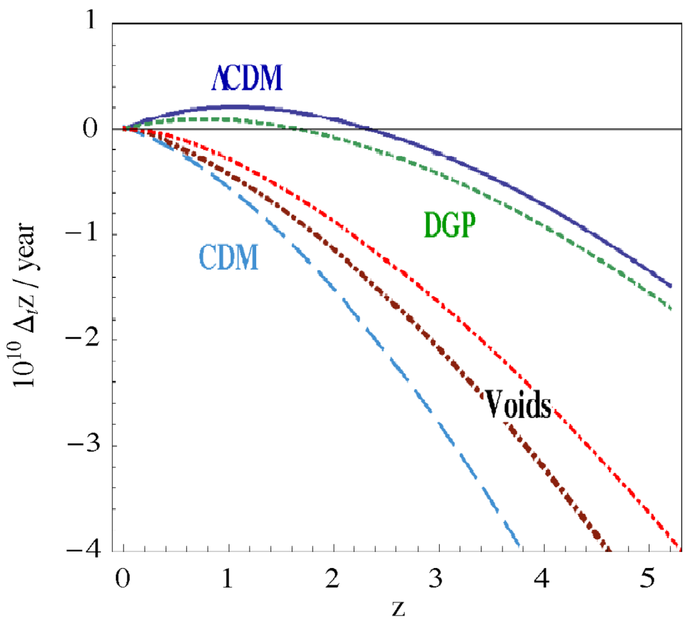
$$\frac{\delta z}{\delta \tau_0} = \frac{\left[\left(\frac{1}{a}\right)^{\cdot} - \frac{1}{4}\left(\frac{k}{a}\right)^{\cdot} r^2\right]_e}{\left[1+\frac{1}{4}kr^2\right]_e} a(\tau_e) - \frac{\left[\left(\frac{1}{a}\right)^{\cdot} + \frac{1}{4}\left(\frac{k}{a}\right)^{\cdot} r^2\right]_o}{\left[1+\frac{1}{4}kr^2\right]_o} a(\tau_0)(1+z) \tag{32}$$

For the model with $(k/a)^{\cdot} = 0$ we have

$$\frac{\delta z}{\delta \tau} = -\frac{H_0}{1+\frac{1}{4}k(\tau_0)r_0^2}\left[\frac{H_e(z)}{H_0} - (1+z)\right] . \tag{33}$$

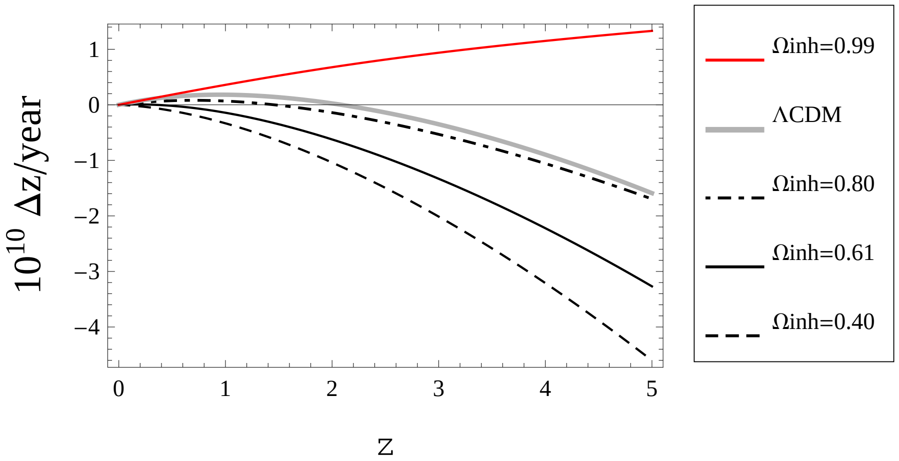
Sandage-Loeb CDM formula for $\Omega_{inh} \to 0$; $H_e(z) = H_0(1+z)^{3/2}$, $r_0 \to 0$.

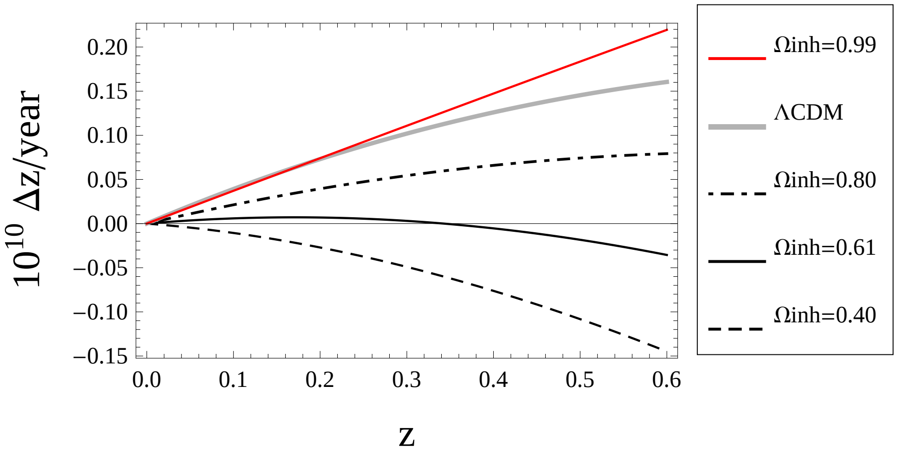
Redshift drift - LTB voids.



- Plots for **3 different LTB** void models, ΛCDM, brane DGP, Cold Dark Matter (CMD) (Quercellini et. al, 2012).
- ΛCDM – the drift is **positive at small redshift**, but becomes negative for $z \gtrsim 2$.
- Giant void (LTB) model mimicking dark energy - the drift is **always negative**.

Redshift drift - inhomogeneous pressure models ($r_0 = 0$, $w = 0$).





- Ω_{inh} **small** - drift as in LTB and CDM models
- Ω_{inh} **larger** - drift as in ΛCDM models (first positive, then negative), e.g. for $\Omega_{inh} = 0.61$ drift is positive for $z \in (0, 0.34)$.
- Ω_{inh} **very large** - drift positive ($\Omega_{inh} = 0.99$ up to $z = 17$; $\Omega_{inh} = 1$ (inhomogeneity-domination) $z > 0$) and $\frac{\delta z}{\delta t} = H_0 \frac{z}{2}$ which means that the drift **grows linearly** with redshift.

Redshift drift - observational perspective.

- One is able to **differentiate** between the drift in ΛCDM models, in LTB models, and in Stephani models - this can be done in future experiments.
- At larger $z > 1.7$ redshifts by **giant telescopes**: European Extremely Large Telescope (E-ELT) with spectrograph CODEX (COsmic Dynamics EXperiment); Thirty Meter Telescope (TMT); Giant Magellan Telescope (GMT).
- At smaller (even $z \sim 0.2$) redshifts by **space-borne gravitational wave interferometers** like DECIGO/BBO (DECi-hertz Interferometer Gravitational Wave Observatory/Big Bang Observer). **This could clearly reject LTB models if the drift measured was positive!**

4. Off-center observer's position against Union2 supernovae.

The luminosity distance is given by

$$d_L = \frac{a_0(1+z)\hat{r}'}{1+\frac{\beta}{4}a_0 r_0^2} \ , \tag{34}$$

with an **off-center** observer placed at r_0, θ_0, ϕ_0 as meant in the coordinate system $\{t, r, \theta, \varphi\}$ of the Stephani metric. More precisely we have

$$d_L = \frac{(1+z)}{1-\frac{a_0 H_0^2 \Omega_{inh}}{4} r_0^2} \ \hat{r}'(\Omega_{inh}, w, r_0, \theta_0, \varphi_0, H_0, \hat{\theta}', \hat{\varphi}', z) \ , \tag{35}$$

where

$$\hat{r}' = \hat{r}'(a) = \frac{1}{H_0} \int_{a_e}^{1} \frac{dx}{\sqrt{(1-\Omega_{inh})x^{1-3w} + \Omega_{inh} x^3}} \ , \tag{36}$$

and a_e is the value of the scale factor at the moment of an emission of the light ray.

Off-center observers

For the redshift one takes

$$1 + z = \frac{a_0(4 - a_e H_0^2 \Omega_{inh} r_e^2)}{a_e(4 - a_0 H_0^2 \Omega_{inh} r_0^2)} \,, \tag{37}$$

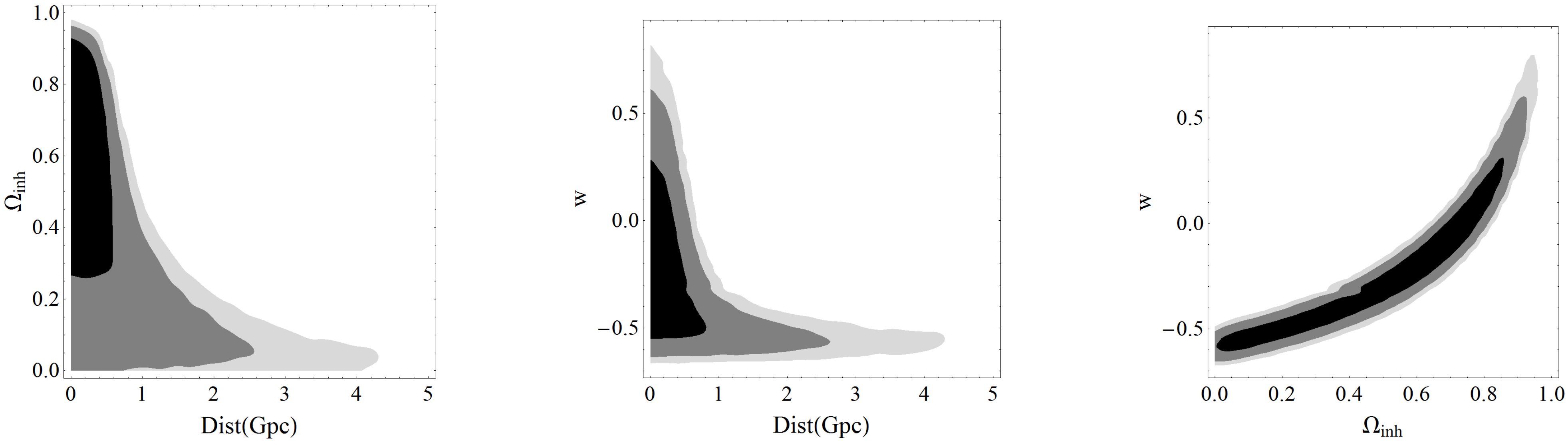
where

$$\begin{aligned} r_e^2 &= (r_0 \sin\theta_0 \cos\varphi_0 + \hat{r}'(a) \sin\hat{\theta}' \cos\hat{\varphi}')^2 \\ &+ (r_0 \sin\theta_0 \sin\varphi_0 + \hat{r}'(a) \sin\hat{\theta}' \sin\hat{\varphi}')^2 \\ &+ (r_0 \cos\theta_0 + \hat{r}'(a) \cos\hat{\theta}' \sin\hat{\varphi}')^2 \end{aligned} \tag{38}$$

and $\hat{\theta}'$ and $\hat{\varphi}'$ are the coordinates of a supernova as seen by an off-center observer in the sky.

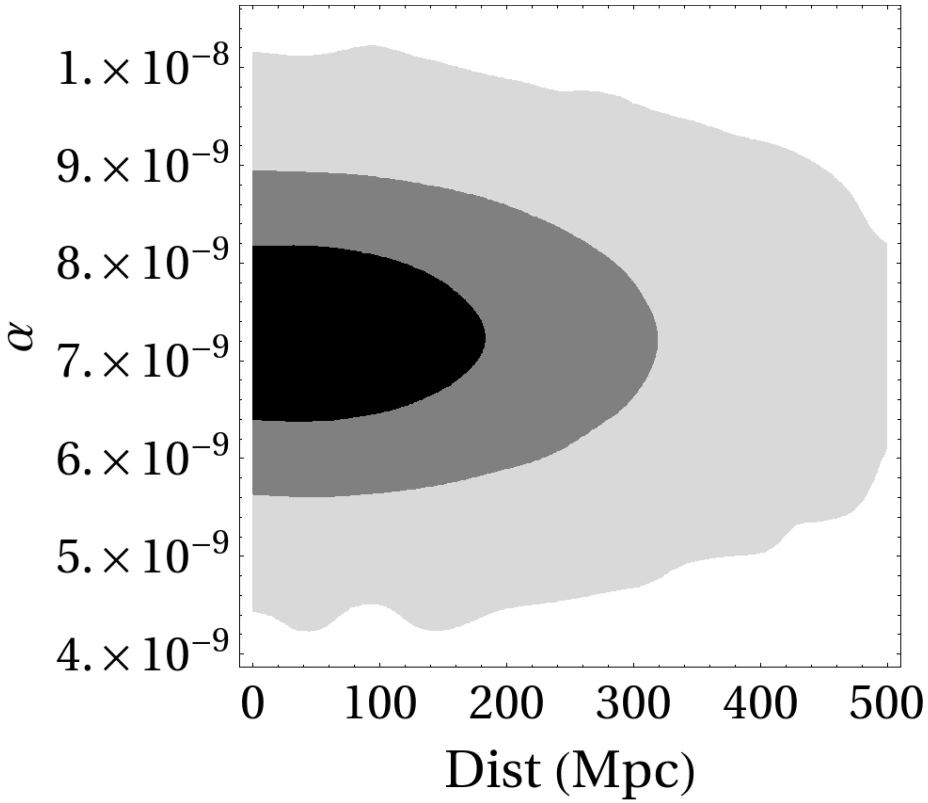
We applied **Union2 557 supernovae data** of Amanullah et al. (2010, ApJ, 716, 712) - we note the courtesy of M. Kowalski and U. Feindt to consult the sample.

Off-center observers - model IIA



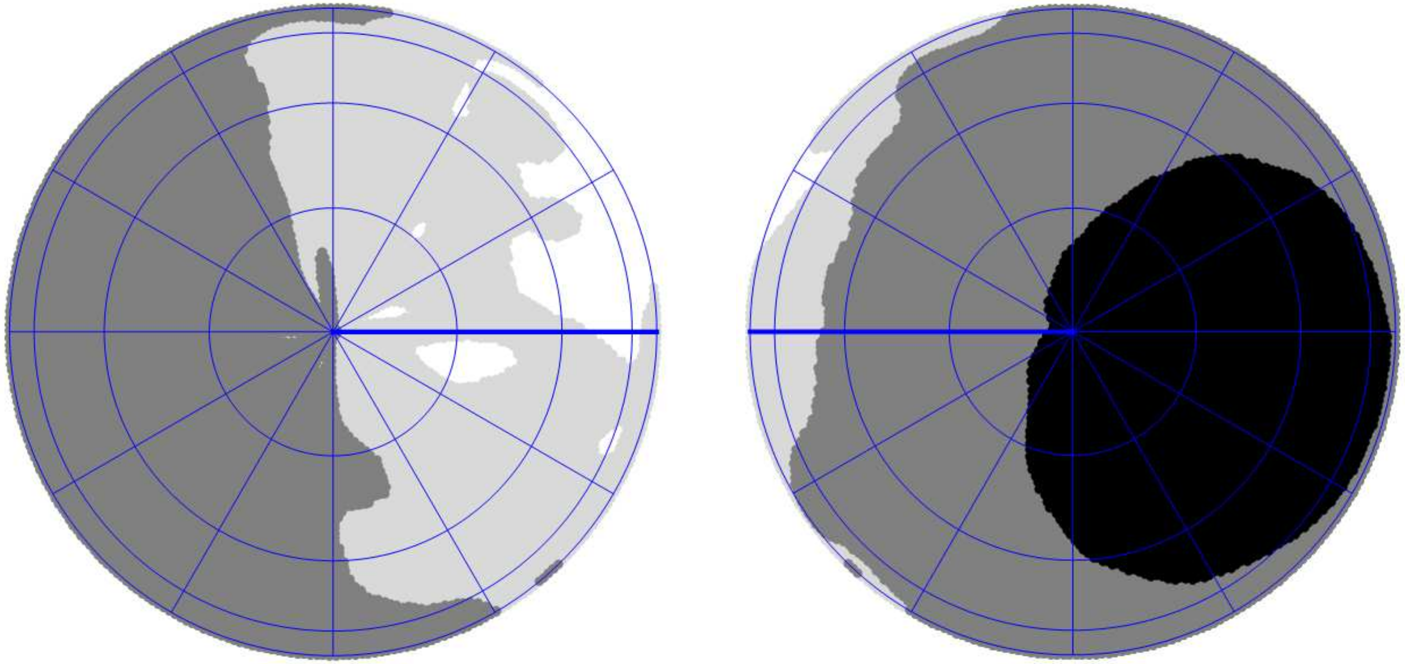
Best-fit values: inhomogeneity density $\Omega_{inh} \sim 0.77$, center of symmetry equation of state barotropic index $w \sim 0.093$, off-center observer position $Dist = 341$ Mpc $(\chi^2 = 526)$.

Off-center observers - model IIB



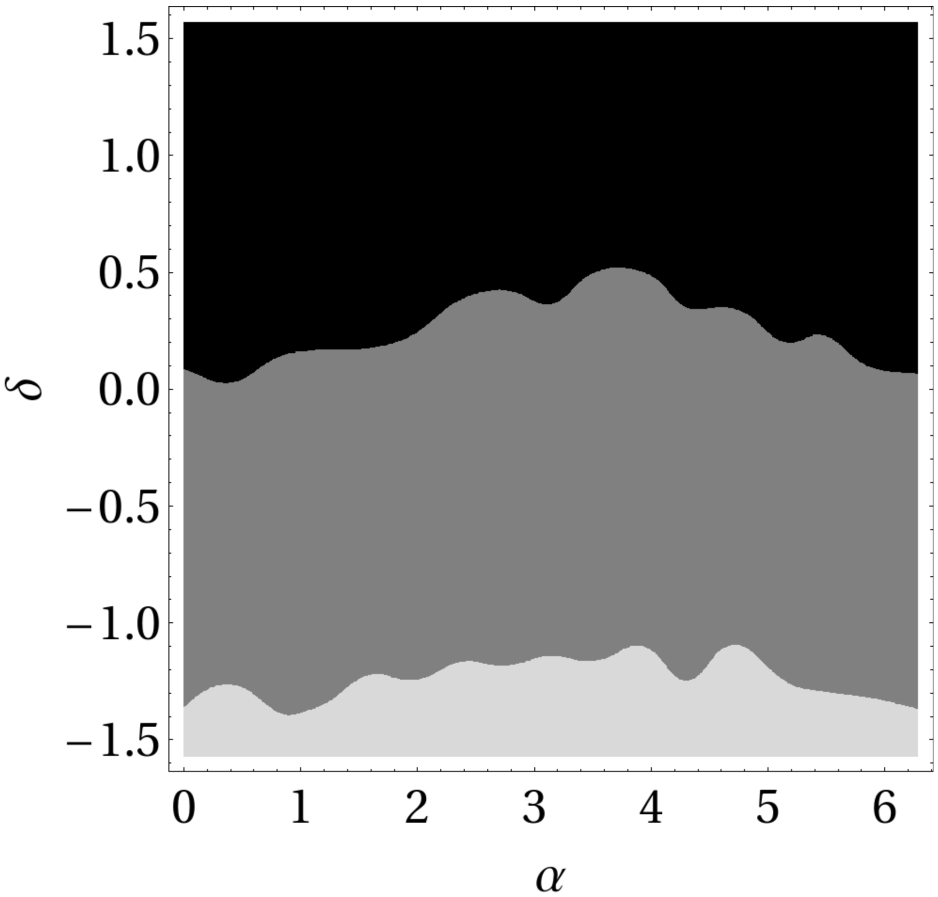
Non-barotropic EOS **limits stronger** the position of an observer. **Best-fit values**: inhomogeneity parameter $\alpha = 7.31 \cdot 10^{-9}\ (s/km)^{2/3} Mpc^{-4/3}$, off-center observer position $Dist =$ 68 Mpc $(\chi^2 = 615)$ ($\alpha = 0$ case (dust) excluded.

Position of the center of symmetry (inhomogeneity)- model IIA



Best fit position: declination $\delta = -65.75^\circ$ and R.A. is $a = 187.33^\circ$. North Celestial Hemisphere (left), South Celestial Hemisphere (right), Meridian line ($\delta = 0$) in bold. In galactic coordinates: $(l, b) = (300.66^\circ, -2.98^\circ)$.

Position of the center of symmetry (inhomogeneity)- model IIB



Best fit position: $\delta = 69.35^\circ$, $a = 8.39^\circ$. In galactic coordinates: $(l, b) = (121.35^\circ, 6.53^\circ)$.

Dipole puzzles:

- Dark flow direction (Watkins et al. 2009) at $(l, b) = (287 \pm 9, 8 \pm 6)$
- Dark energy dipole (Mariano et al. 2012) at $(l.b) = (309, -15)$
- Fine structure α dipole (Webb et al. 2011) at $(l, b) = (320, -11)$
- kSZ effect on CMB (Kashlinsky et al. 2010) at $(l, b) = (296 \pm 13, 140 \pm 13)$
- Dark flow direction (Turnball et al. 2012) at $(l, b) = (319 \pm 18, 70 \pm 14)$

5. More tests in progress - central observer (BAO, CMB shift parameter).

The luminosity distance for a central observer $r_0 = 0$ is (same as Friedmann)

$$d_L = (1+z)a_0 r \ , \tag{39}$$

and the distance modulus is

$$\mu(z) = 5\log_{10} d_L(z) + 25. \tag{40}$$

From the null geodesic equations we have (model IIA)

$$r = c\int_a^{a_0} \frac{da}{\sqrt{c^2A^2a^{1-3w} - \beta c^2 a^3}} = r = \frac{c}{H_0 a_0}\int_{a/a_0}^{1} \frac{dx}{\sqrt{\Omega_0 x^{1-3w} + (1-\Omega_0)x^3}} \ , \tag{41}$$

where $x \equiv a/a_0$. Using the definition of redshift (29) one can rewrite (41) as

$$z(x) = \frac{1}{x} - 1 + \frac{\Omega_0 - 1}{4}\left[\int_{a/a_0}^{1} \frac{dx}{\sqrt{\Omega_0 x^{1-3w} + (1-\Omega_0)x^3}}\right]^2 \ , \tag{42}$$

Combined tests - luminosity distance, apparent magnitude

and so the luminosity distance (39) reads as

$$d_L(x) = \frac{c(1+z)}{H_0}\sqrt{\frac{4[z(x)+1-1/x]}{\Omega_0 - 1}} \quad . \tag{43}$$

CMB shift parameter.

The shift parameter is defined as:

$$\mathcal{R} = \frac{l_1'^{TT}}{l_1^{TT}} \quad , \tag{44}$$

where l_1^{TT} – the temperature perturbation CMB spectrum multipole of the first acoustic peak in inh. pressure model
$l_1'^{TT}$ – the multipole of a reference flat standard Cold Dark Matter model. The multipole number is related to an angular scale of the sound horizon r_s at decoupling by

$$\theta_1 = \frac{r_s}{d_A} \propto \frac{1}{l_1} \ . \tag{45}$$

For our Stephani model the angular diameter distance is given by

$$d_A = \frac{a_{\rm dec}}{V(t_{dec}, r_{dec})} r_{\rm dec} \tag{46}$$

with $r_{\rm dec}$ given by (41) taken at decoupling.

CMB shift parameter.

Using the above, we may write that for our Stephani models the shift parameter is

$$\mathcal{R} = \frac{2cV(t_{dec}, r_{dec})}{H_0\sqrt{\Omega_0}r_{\rm dec}} \; . \tag{47}$$

Finally, the rescaled shift parameter is

$$\bar{\mathcal{R}} \quad = \quad \frac{H_0\sqrt{\Omega_0}r_{\rm dec}}{cV(t_{dec}, r_{dec})} \; . \tag{48}$$

The WMAP data gives $\bar{\mathcal{R}} = 1.70 \pm 0.03$ (Wang, Mukherjee 2006).

Baryon acoustic oscillations.

One calculates the distortion of a spherical object in the sky without knowing its true size by measuring its transverse extent using the angular diameter distance, r

$$r = \frac{l}{\Delta\theta} \ , \tag{49}$$

where l and $\Delta\theta$ are the linear and angular size of an object, and its line-of-sight extent, Δr, using the redshift distance

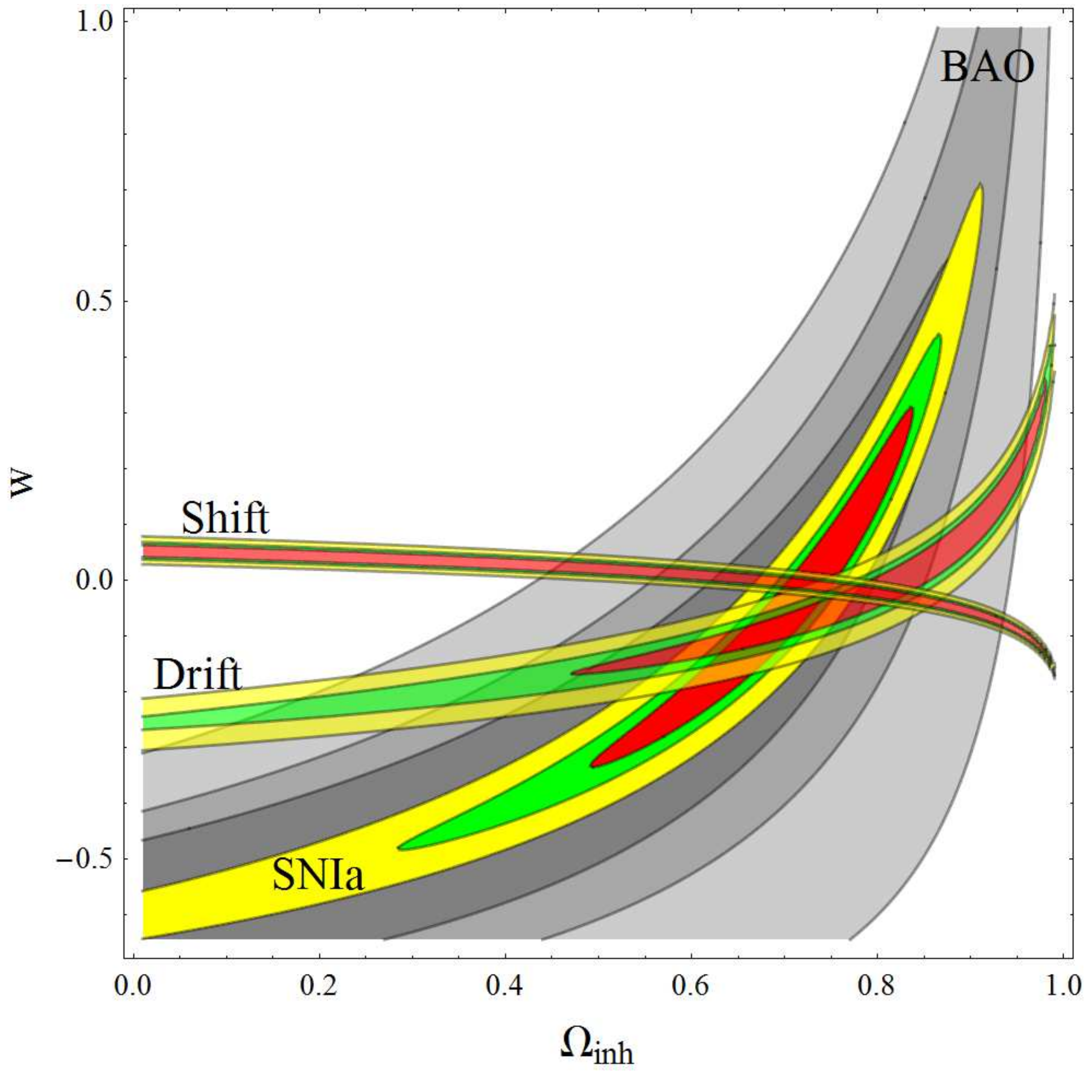
$$\Delta r = \frac{c\Delta t}{a(t)} \tag{50}$$

(see e.g. Nesseris (2006)). As a result one can define the volume distance, D_V, as

$$D_V^3 = r^2 \Delta r \ . \tag{51}$$

Eisenstein et al. (2005) gave $D_V(\Delta z = z_{BAO} = 0.35) = 1370 \pm 64$ Mpc (an acoustic peak for 46748 luminous red galaxies (LRG) selected from the SDSS (Sloan Digital Sky Survey).

Inhomogeneous pressure - combined tests (SNIa, RD, BAO, shift parameter)



Inhomogeneous pressure - combined tests: results and improvements

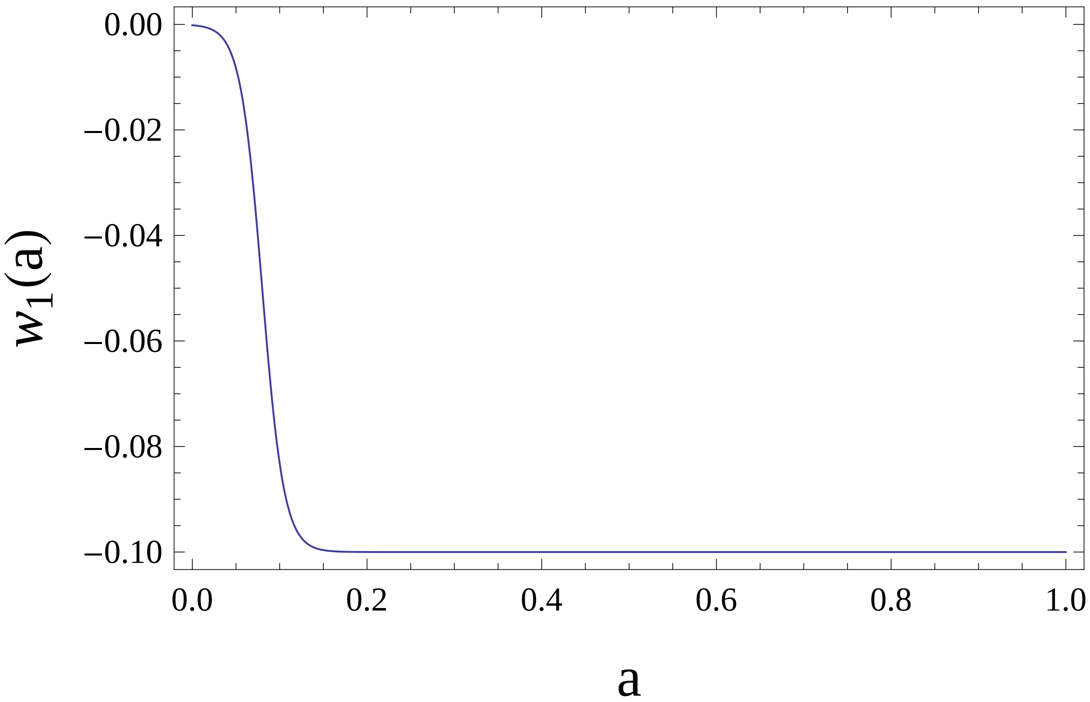
- Stephani model fits well the data for the SNIa, redshift drift, and BAO (contours overlap at 1σ CL).
- It cannot fit the existing observational data and recover at the same time the shift parameter features of the ΛCDM model (at least within 1σ CL).
- Way out: replace constant barotropic index w **by** $w_1(a)$**.**
- One assumes that $w_1(a)$ suddenly changes somewhere between $z = 5$ and z_{dec}, and then remains constant.

Inhomogeneous pressure models - $w_1(a)$ parametrization

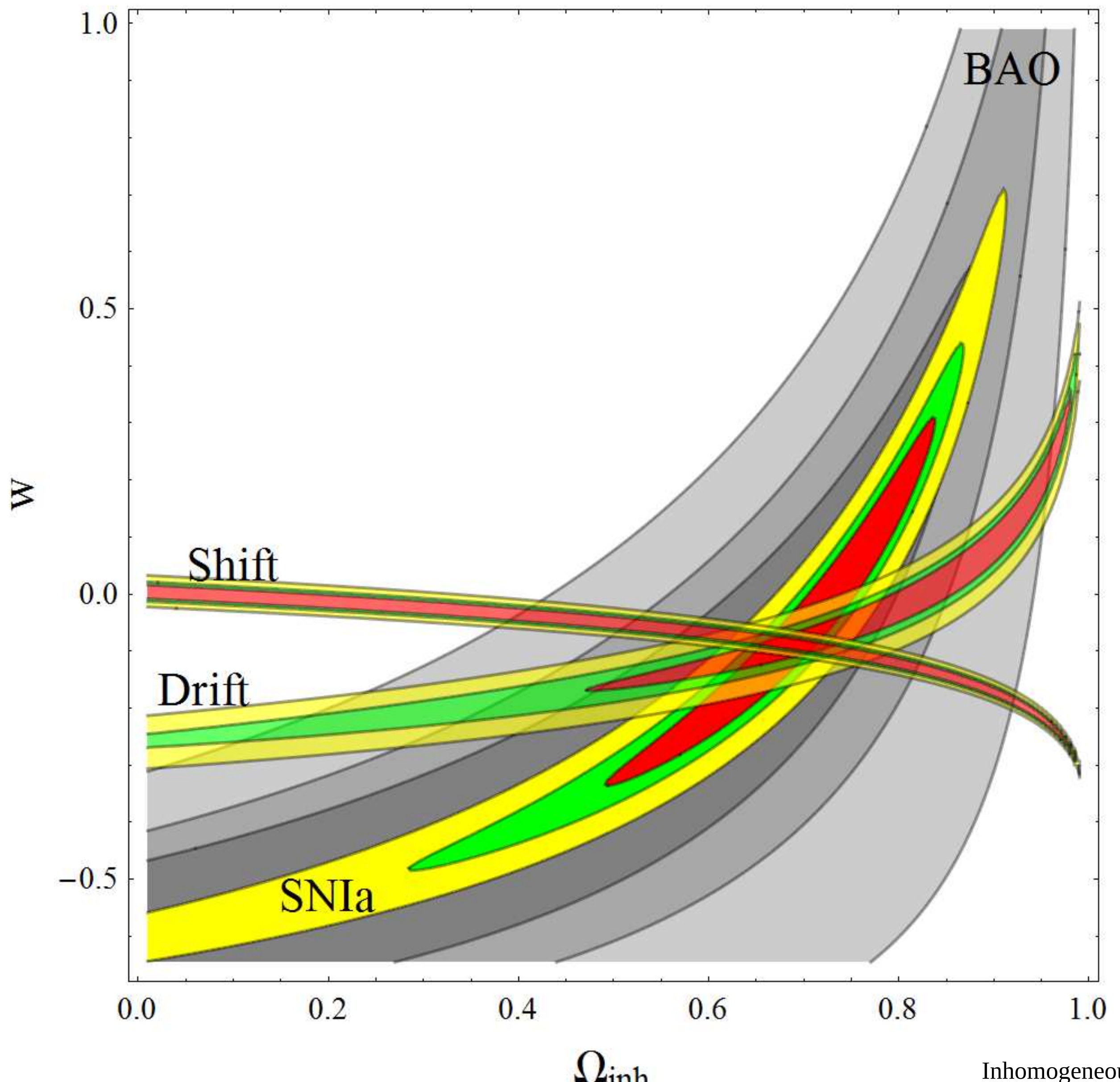
An example of a barotropic index parametrization $w_1(a)$ which can fit the data is:

$$w_1(a) = w + \frac{w_0}{2}\left(1 + \tanh[\lambda(a_{tr} - a)]\right) \ . \tag{52}$$

where w, w_0, λ, and a_{tr} are constants. Here: $\lambda = 40$, $a_{tr} = 0.08$, $w_0 = 0.1$, $w = -0.1$ and $\Omega_{inh} = 0.68$, $z_{tr} \sim 10.66$.



Inhomogeneous pressure models - combined tests for $w_1(a)$



Advantages for future work:

The general inhomogeneous pressure metric

$$ds^2 = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dx^2 + dy^2 + dz^2\right], \quad (53)$$

$$V(t,x,y,z) = 1 + \frac{1}{4}k(t)\left\{[x - x_0(t)]^2 + [y - y_0(t)]^2 + [z - z_0(t)]^2\right\},$$

with x_0, y_0, z_0 being arbitrary functions of time is just a generalization of both the FRW and SS Stephani metrics in isotropic coordinates.

- **It has no symmetries** acting on spacetime.
- The center of symmetry **changes its position** from slice to slice.

6. Conclusions

- Viable and complementary to LTB cosmologies which can **drive acceleration**.
- Clear **difference** against LTB for redshift drift (can be tested by very large telescopes and GW detectors).
- Comparison with the 557 Union2 supernovae data **restricts the position of non-centrally placed observers**. Best fit values are: $Dist = 341 Mpc$ (model IIA) with inhomogeneity density $\Omega_{inh} = 0.77$ (model IIA) and $Dist = 68 Mpc$ with inhomogeneity parameter $\alpha = 7.31 \cdot 10^{-9}$ $(s/km)^{2/3} Mpc^{-4/3}$ (model IIB).
- Gives a dipole which is directed at $(l, b) = (300.66^\circ, -2.98^\circ)$ (model IIA) and $(l, b) = (121.35^\circ, 6.53^\circ)$ (model IIB) and can be compared (if aligned) with other dipoles (dark energy, dark flow, varying-α dipole, etc.)
- Stephani model fits well the data for SNIa, redshift drift, shift parameter, and BAO provided a specific parametrization for $w = w(a)$ is applied.
- Can even model **total spacetime inhomogeneity**.

Thank You!

Physical interpretation of inhomogeneous pressure models

- A fluid with spatially varying equation of state (spatially varying vacuum energy like Λ) is assumed which gives a nongravitational force in the Universe (which manifests as non-zero acceleration of comoving observers).
- Inhomogeneous pressure models can be considered as a kind of interior of a TOV exotic star filled with matter like generalized (anti)-Chaplygin gas $p = \pm A^2/\varrho^\alpha$ ($A =$ const.) (e.g. Kamenschchik et al. 2004, 2008).
- In these models there exists a static spherically symmetric configuration in which the central pressure at $r = 0$ was constant, while on some shell of constant radius r_s it became minus infinity (which is an analogue of a FD singularity). Everywhere between $r = 0$ and $r = r_s$, the pressure is lower than at the center, so that the particles are accelerated away which is exactly the effect which is present in the inhomogeneous pressure model.
- There is also an ideal gas interpretation of these models (Sussmann 2000).