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Nonlinear effects of general relativity from multi-scale structure

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The work is a part of the project

The role of the small-scale inhomogeneities in general relativity and cosmology

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NON-LINEARITY OF GR

Einstein equations (GR)

$$G_{\mu\nu}[g_{\alpha\beta}] = 8\pi G\, T_{\mu\nu}$$

non-linear

Poisson equation (Newtonian)

$$\Delta\phi = 4\pi G\rho$$

linear

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linearization

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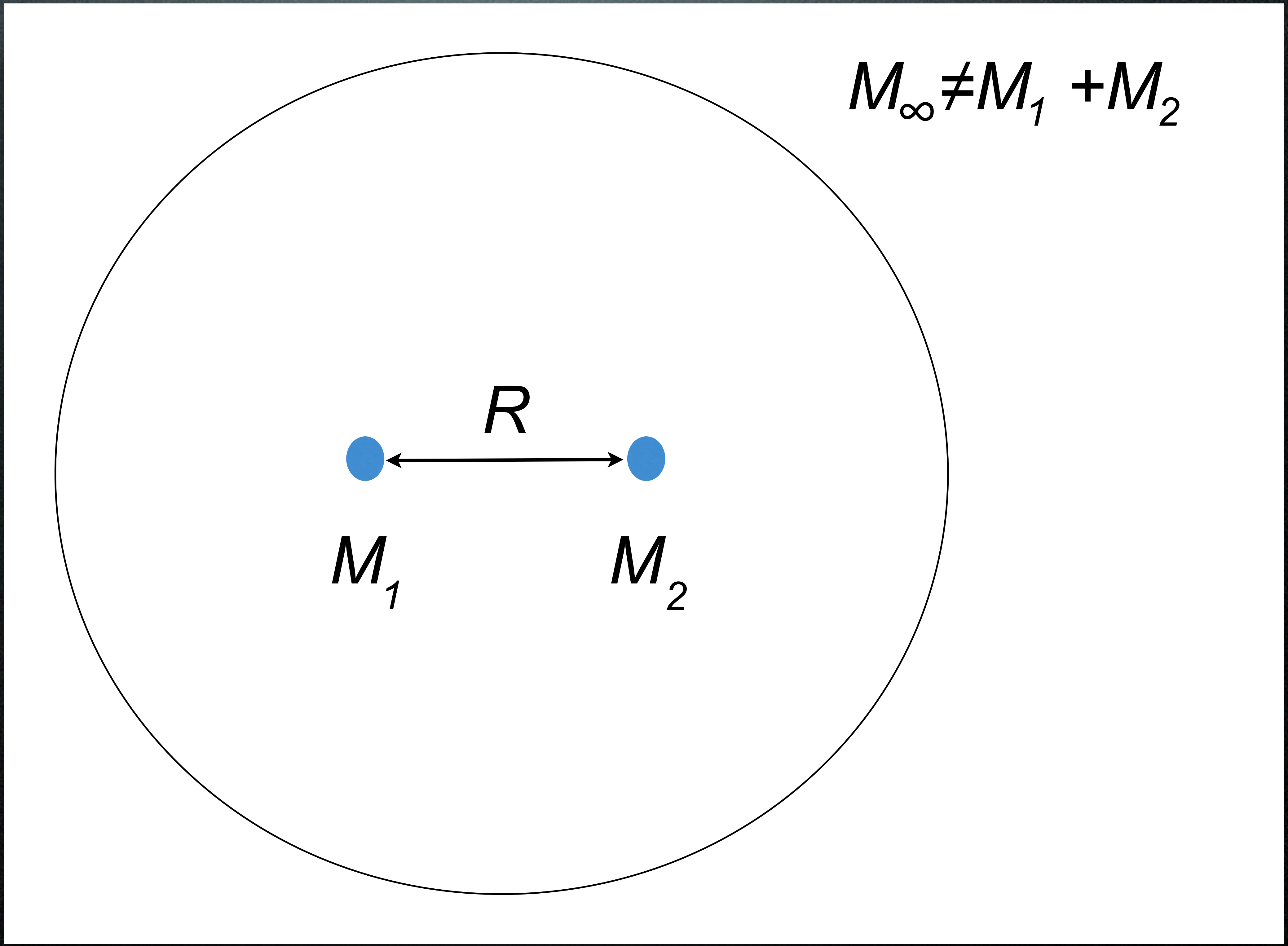
non-linear → linear

linearization

Backreaction problem in cosmology: do we need anything beyond the 1st order perturbation for the cosmic structure?

Non-linearity of GR



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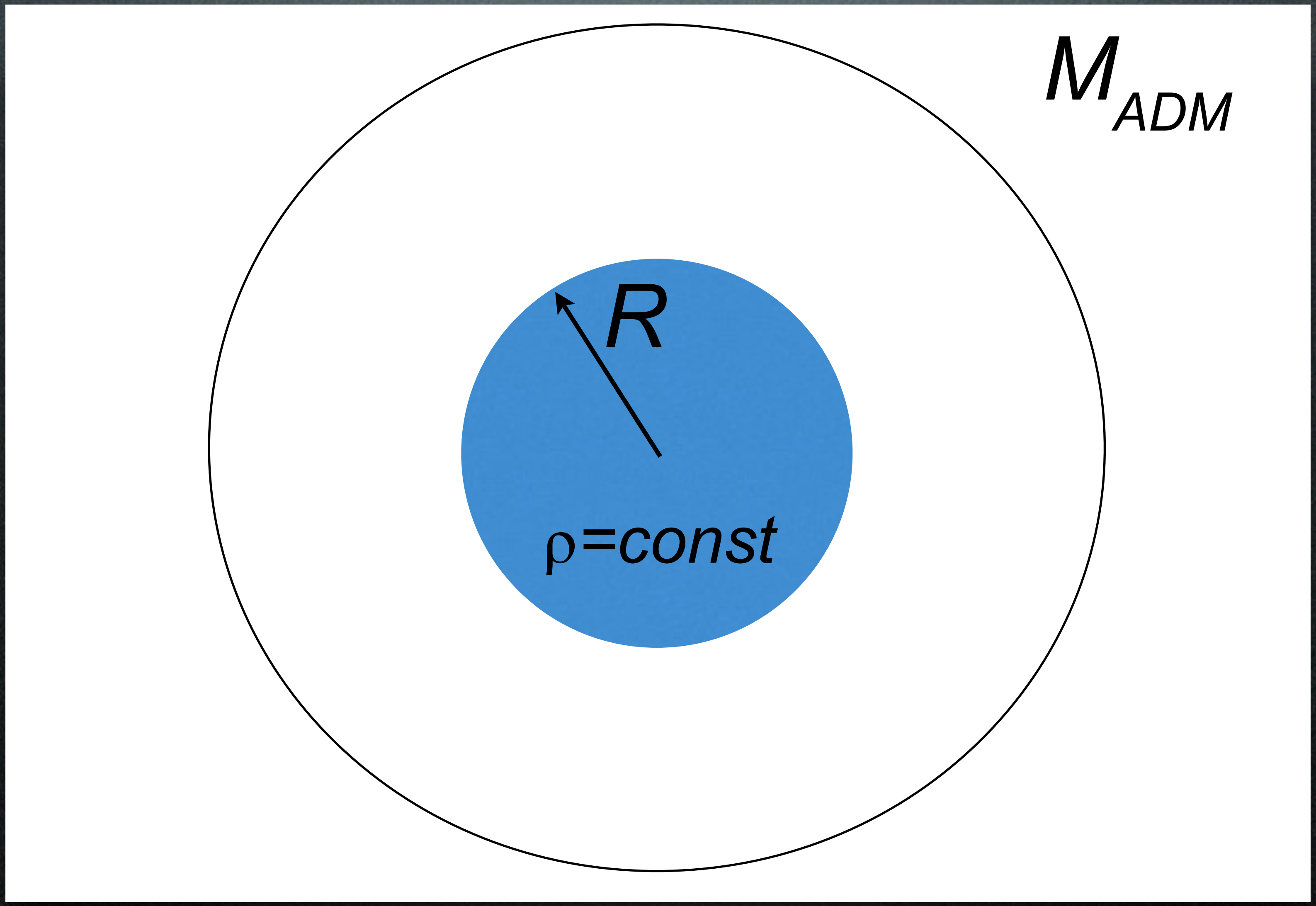
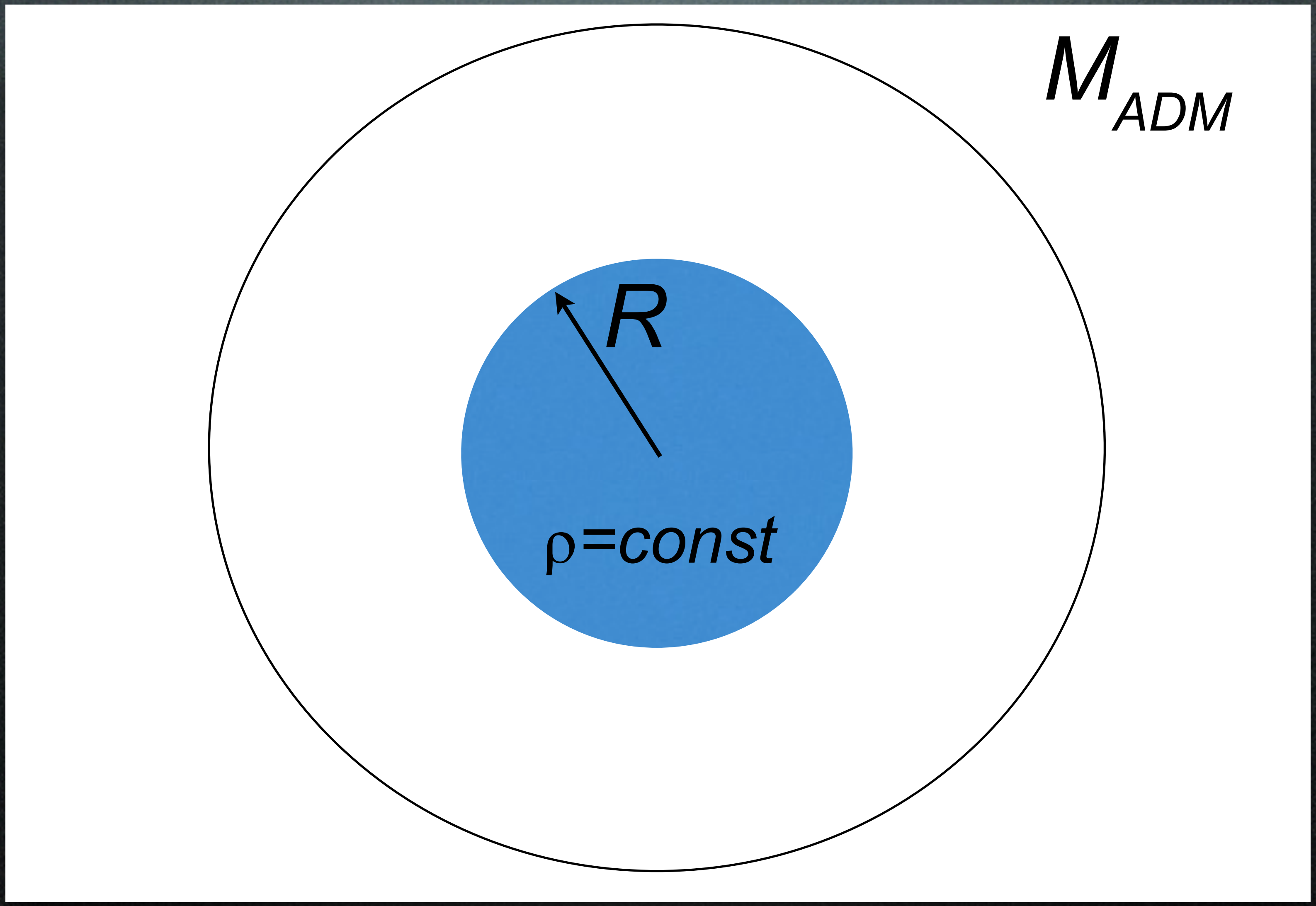


$$M_{tot} = \int \rho \, \mathrm{d}V \neq M_{ADM}$$

NON-LINEARITY OF GR



$$x = \frac{M_{tot} - M_{ADM}}{M_{ADM}} = \frac{3}{5}\varepsilon + O(\varepsilon^2)$$

$$\varepsilon = \frac{GM}{R}$$

NON-LINEARITY OF GR

Earth: $\varepsilon = 1.4 \cdot 10^{-9}$

$$\varepsilon = \frac{GM}{R}$$

Sun: $\varepsilon = 2 \cdot 10^{-8}$

Galaxy: $\varepsilon = 10^{-4} - 10^{-6}$

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Does small ϵ imply small corrections?

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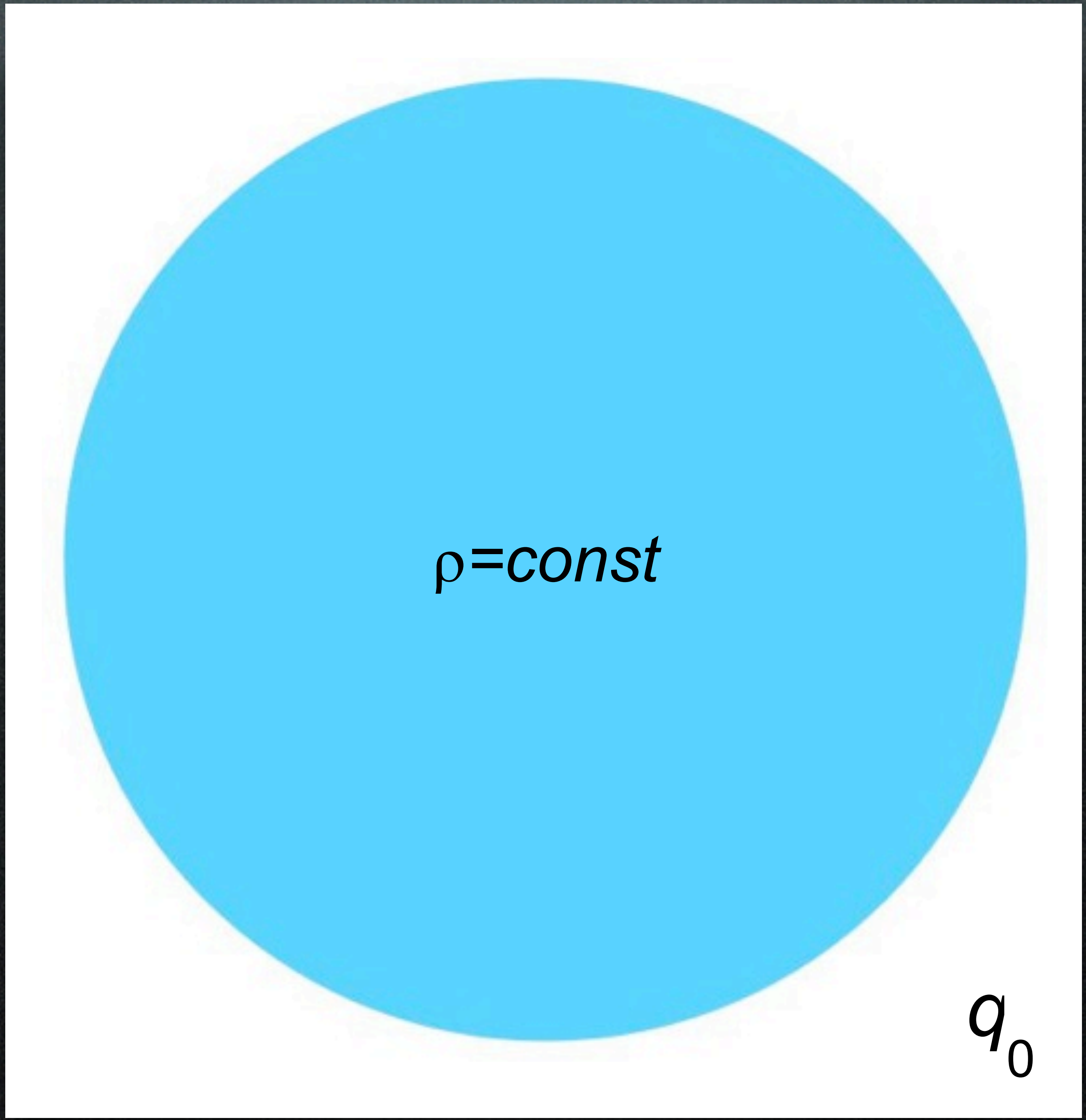
Does small ϵ imply small corrections?

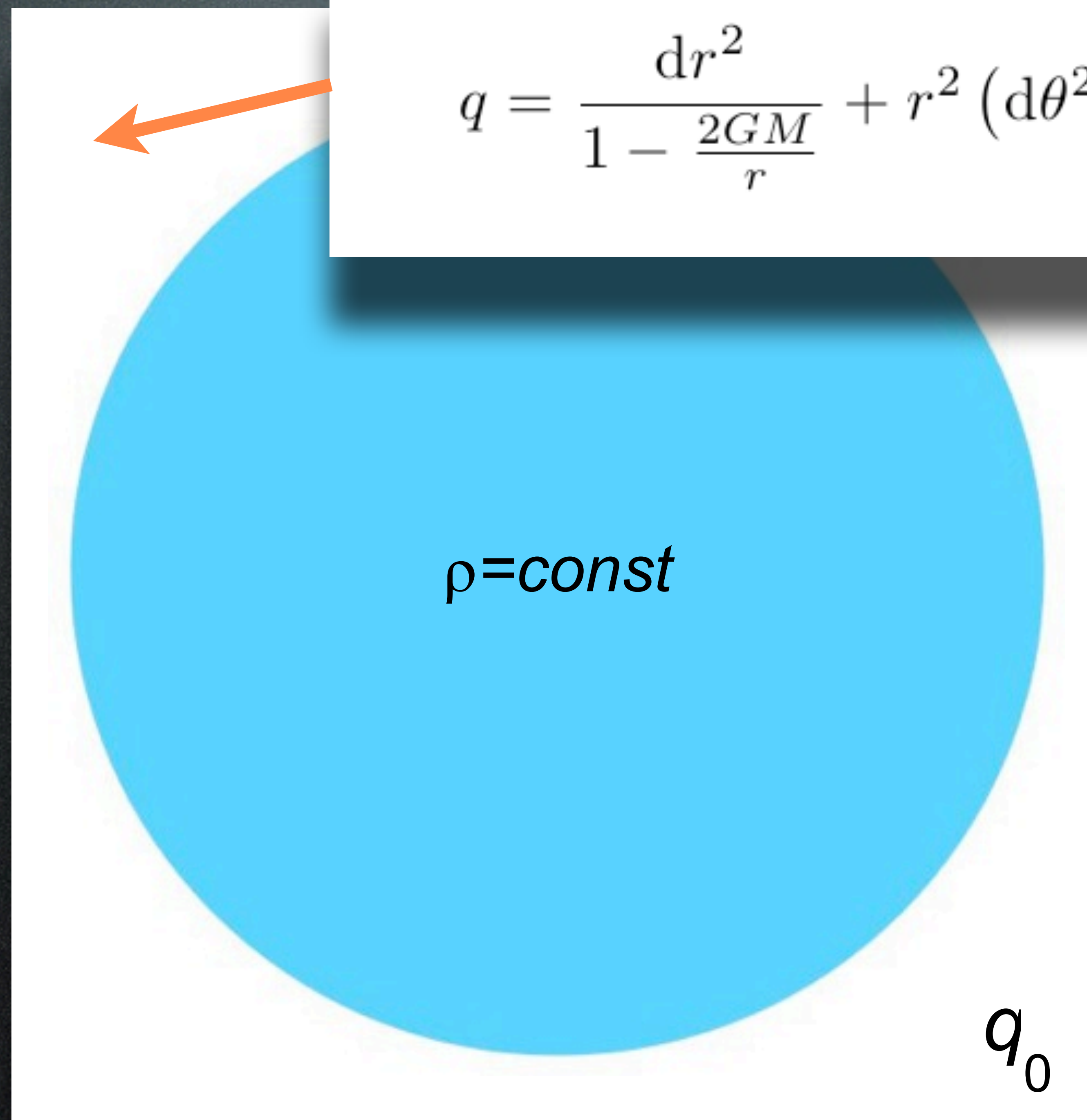
Not always: a counterexample!

Multiscale foam solution



MULTISCALE FOAM SOLUTION

$$q = \frac{\mathrm{d}r^2}{1 - \frac{2GM}{r}} + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\varphi^2\right)$$



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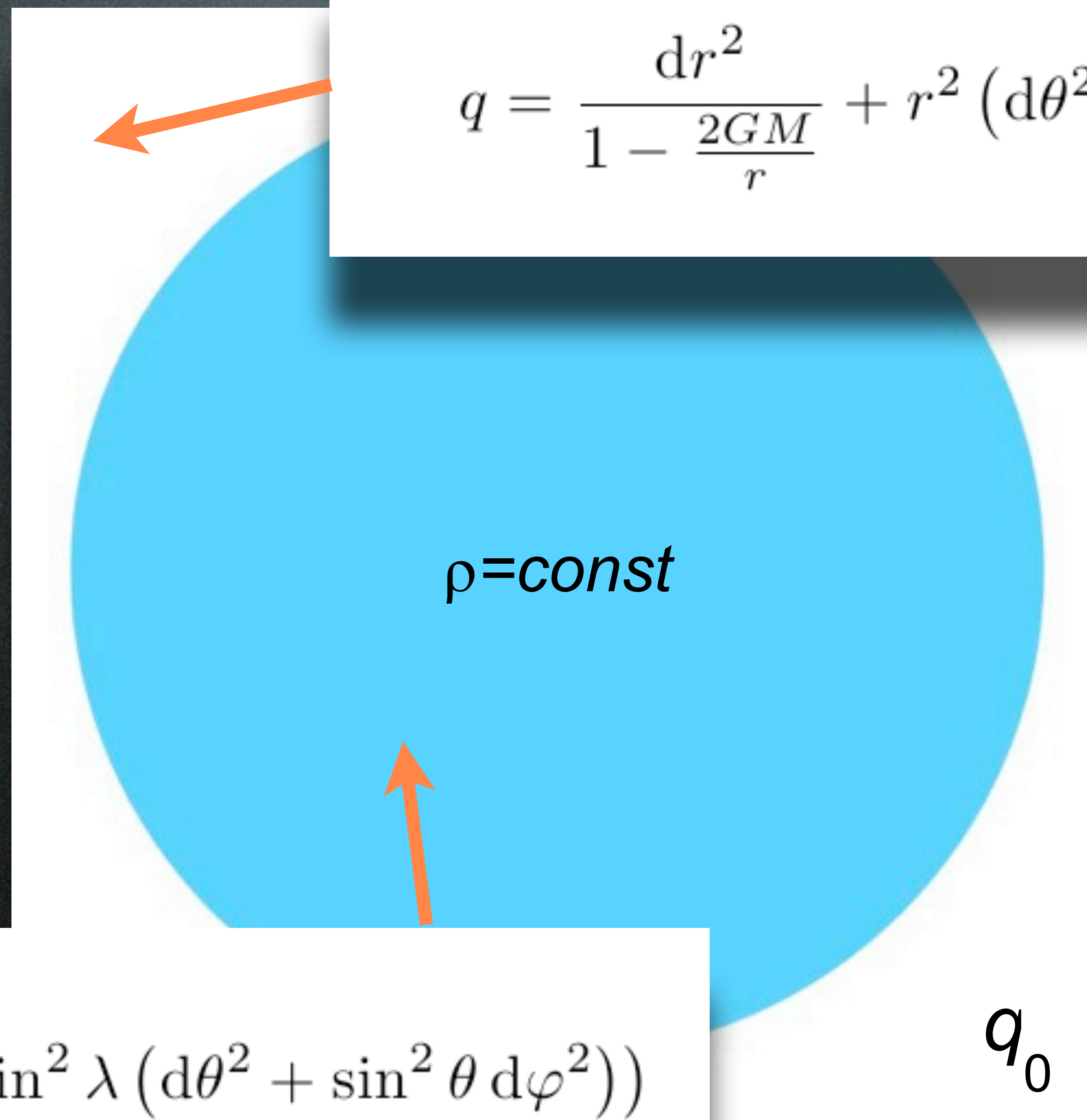
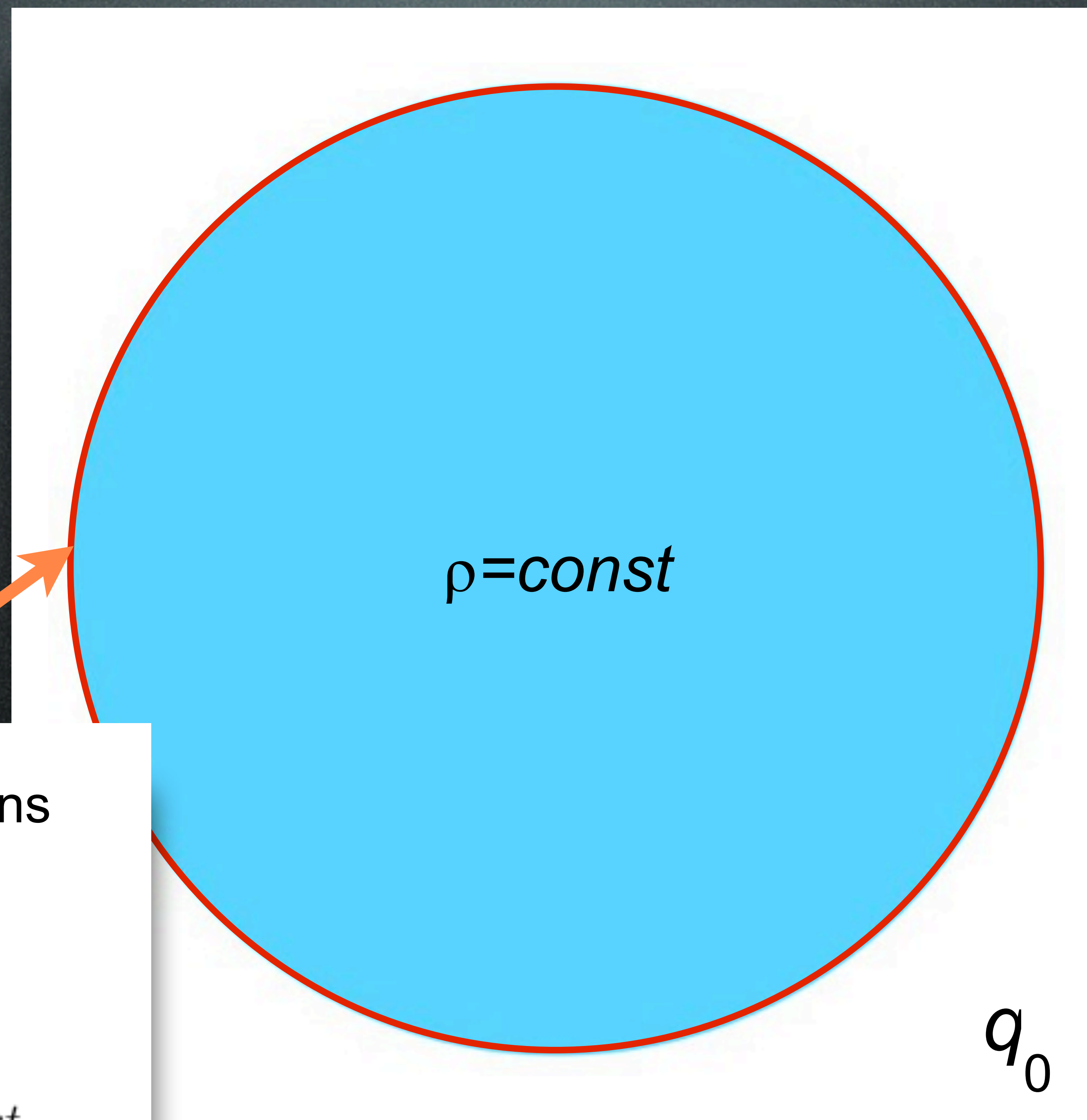


$$q = \mathcal{R}^2 \left(\mathrm{d}\lambda^2 + \sin^2\lambda \left(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\varphi^2\right)\right)$$

Multiscale foam solution

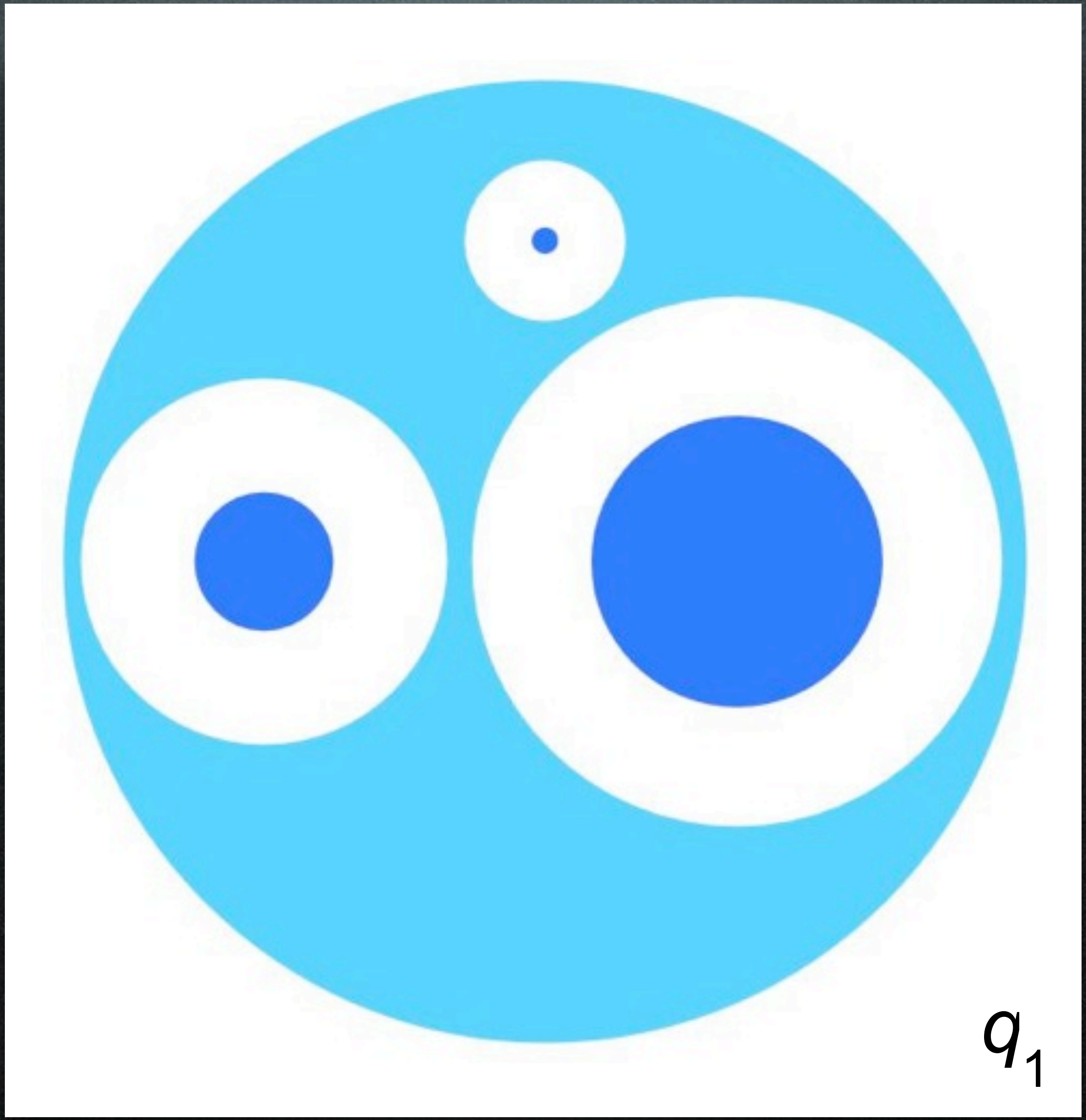


matching conditions

$$A_{int} = A_{ext}$$

$$\left.\frac{\mathrm{d}A}{\mathrm{d}n}\right|_{int} = \left.\frac{\mathrm{d}A}{\mathrm{d}n}\right|_{ext}$$

Multiscale foam solution



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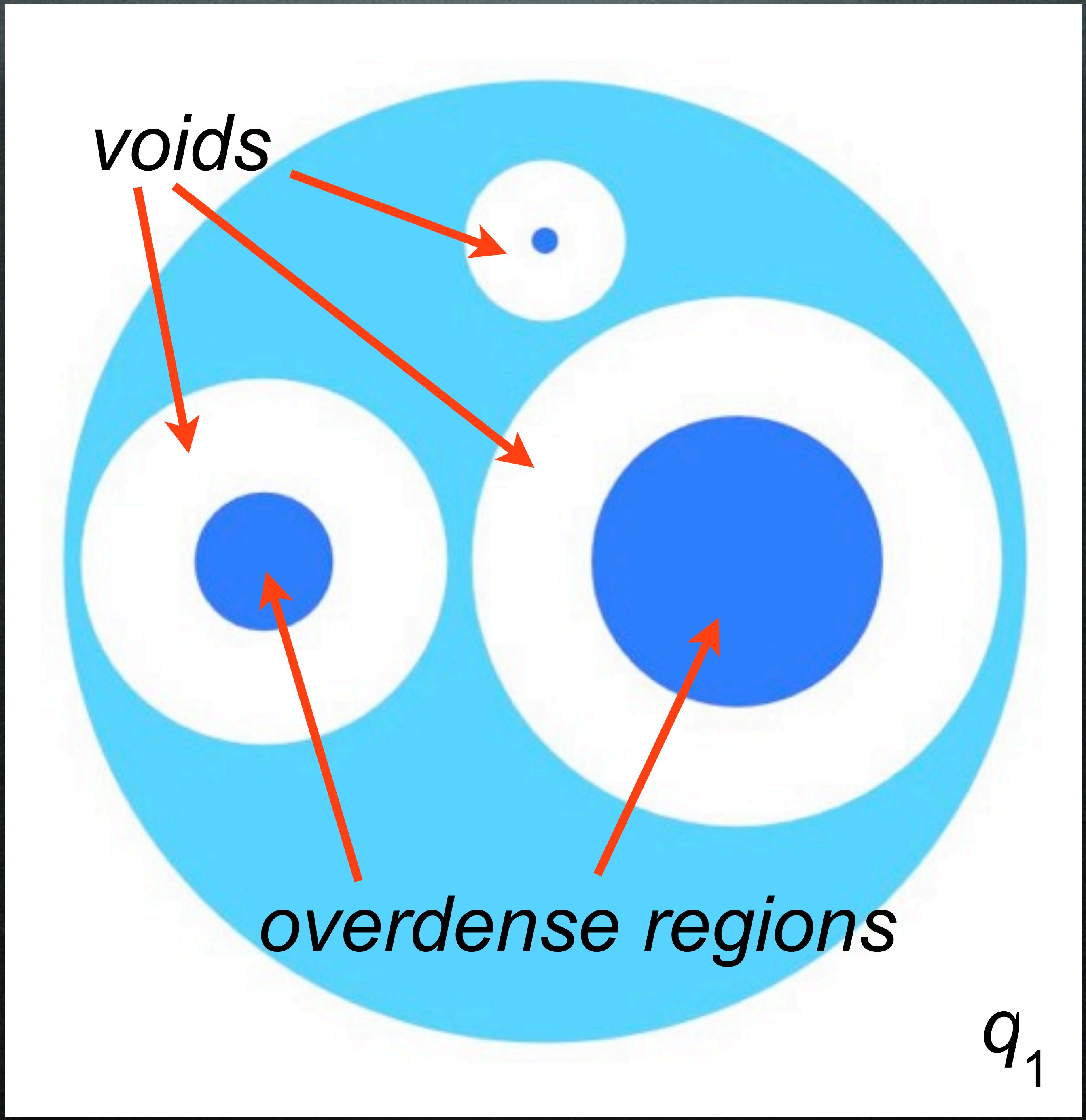

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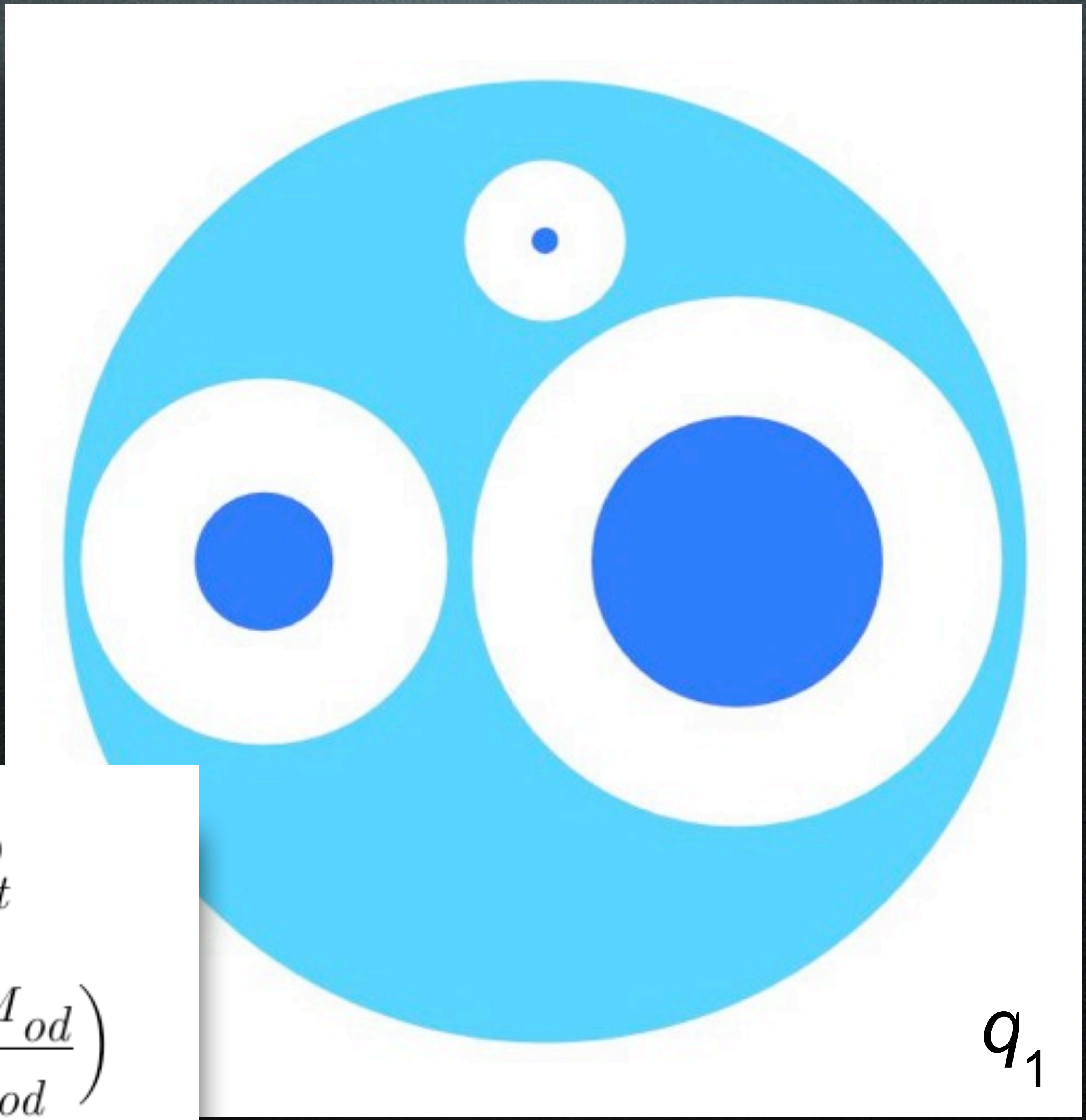
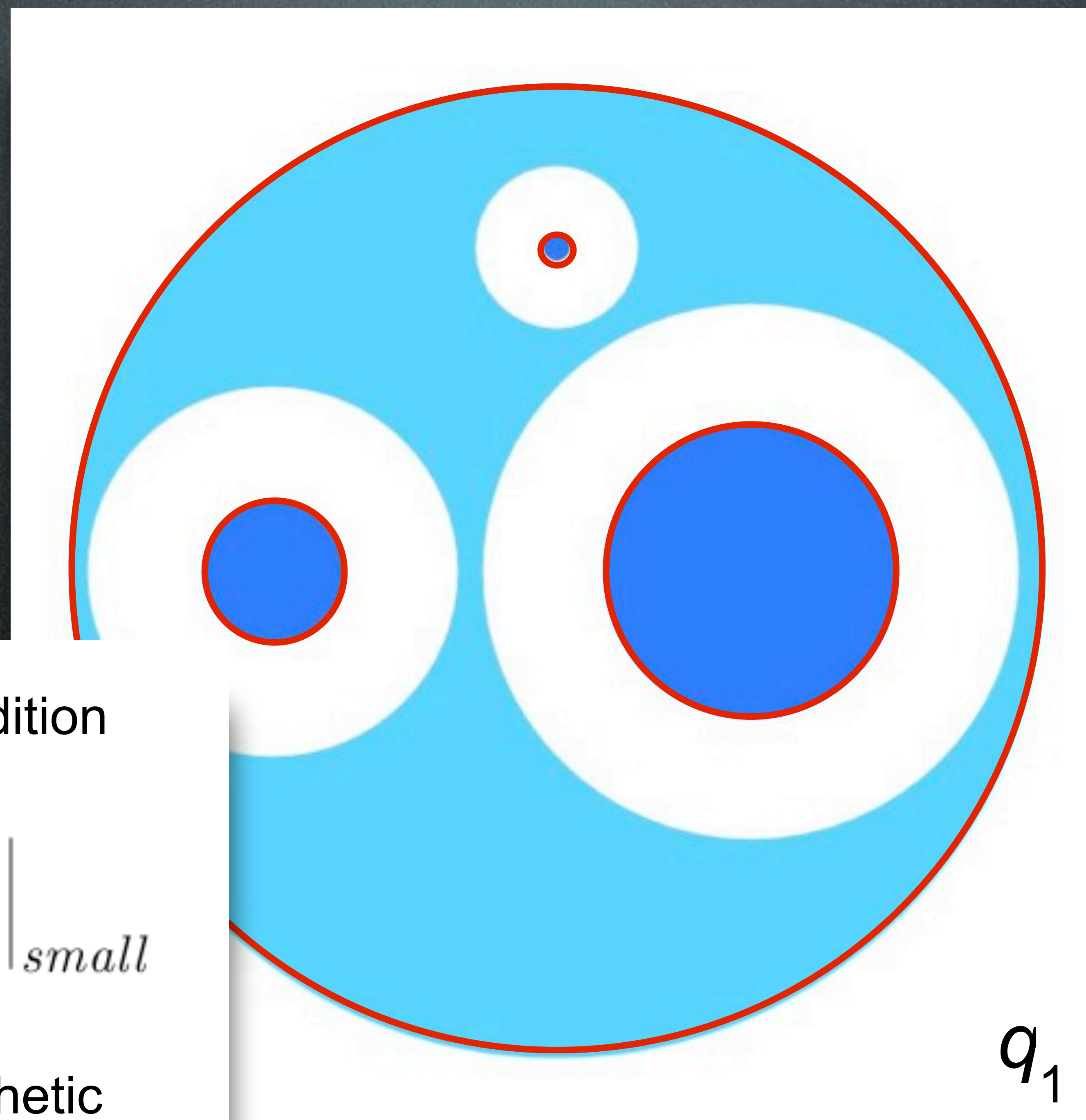


$$M_{tot} < M_{tot}^{(1)}$$

$$\Delta M_{tot}^{(1)} = O\left(\frac{GM_{od}}{R_{od}}\right)$$

MULTISCALE FOAM SOLUTION

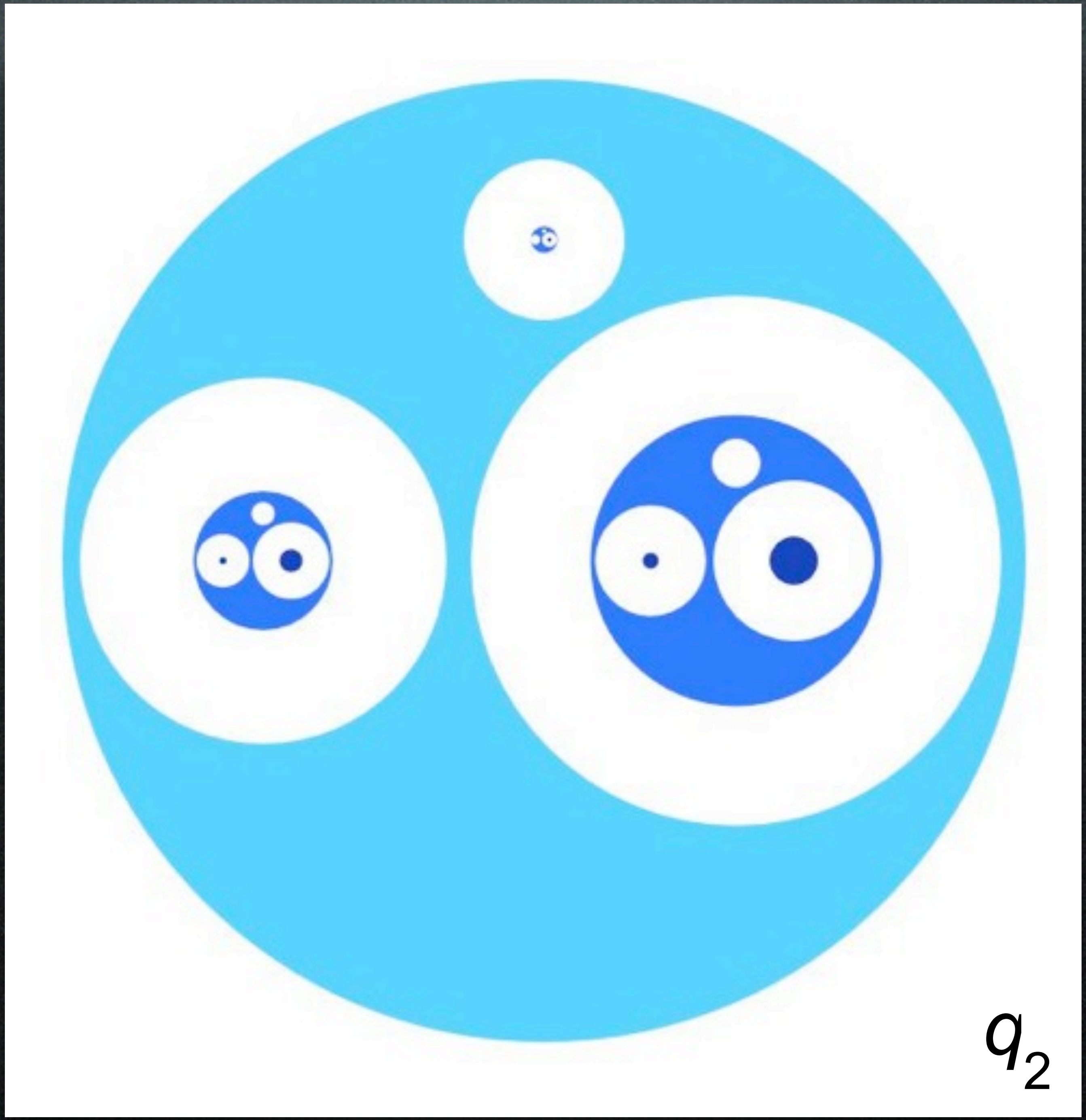


additional condition

$$\left.\frac{\mathrm{d}\sqrt{A}}{\mathrm{d}n}\right|_{big} = \left.\frac{\mathrm{d}\sqrt{A}}{\mathrm{d}n}\right|_{small}$$

interiors homothetic

MULTISCALE FOAM SOLUTION



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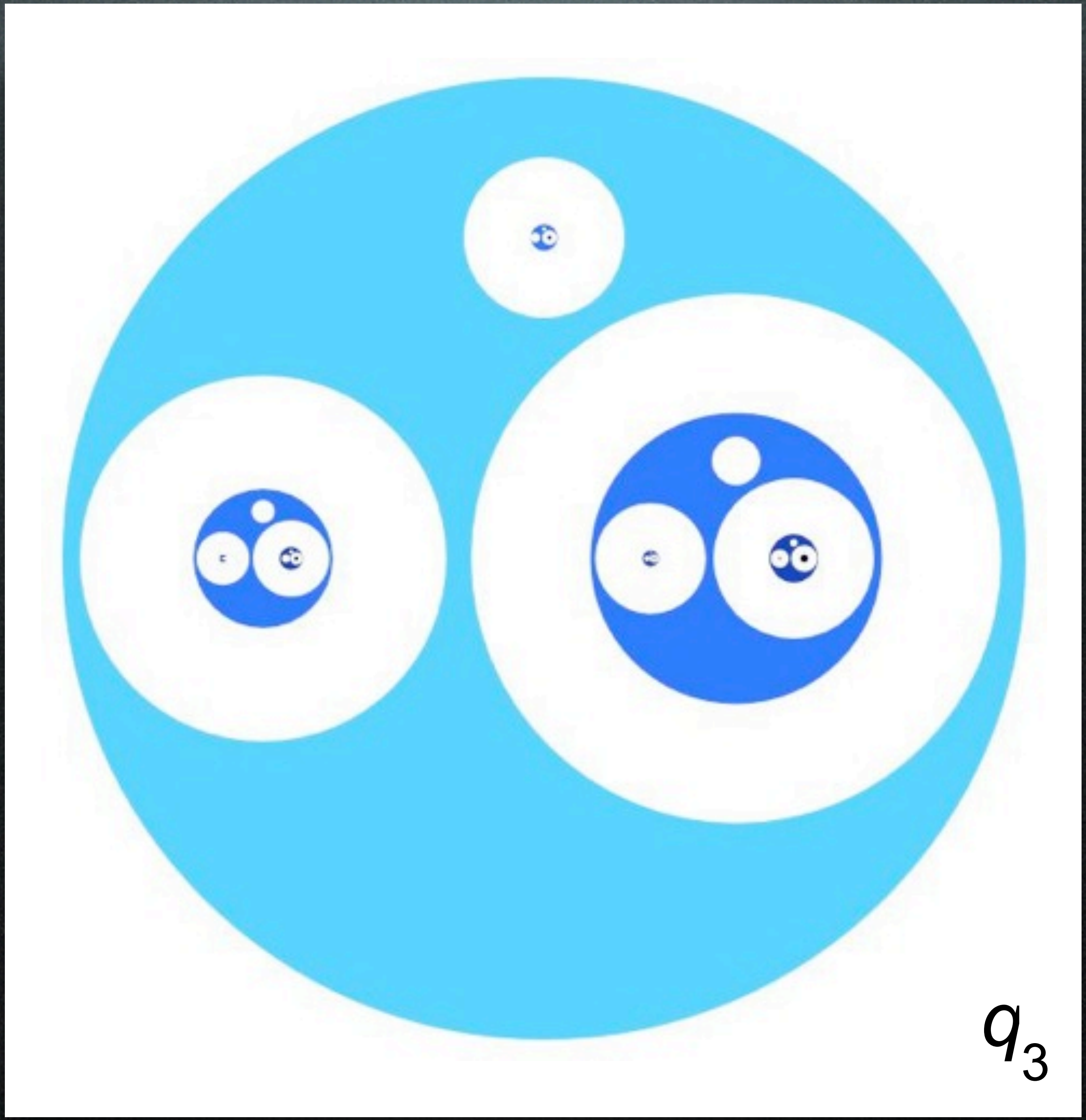

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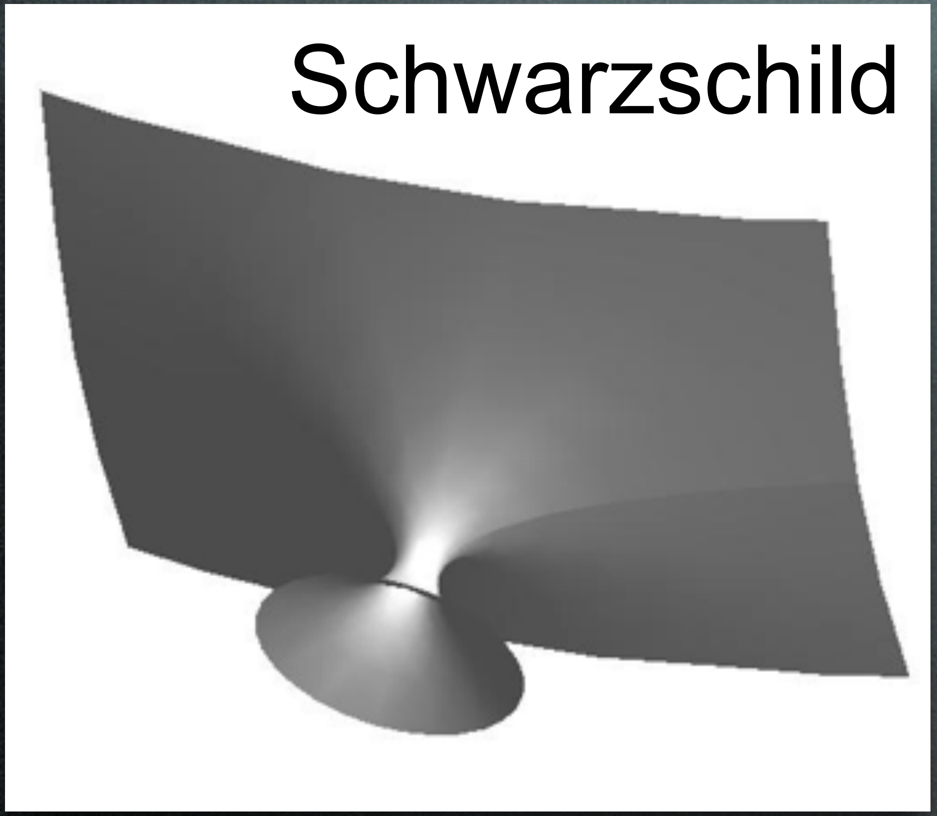

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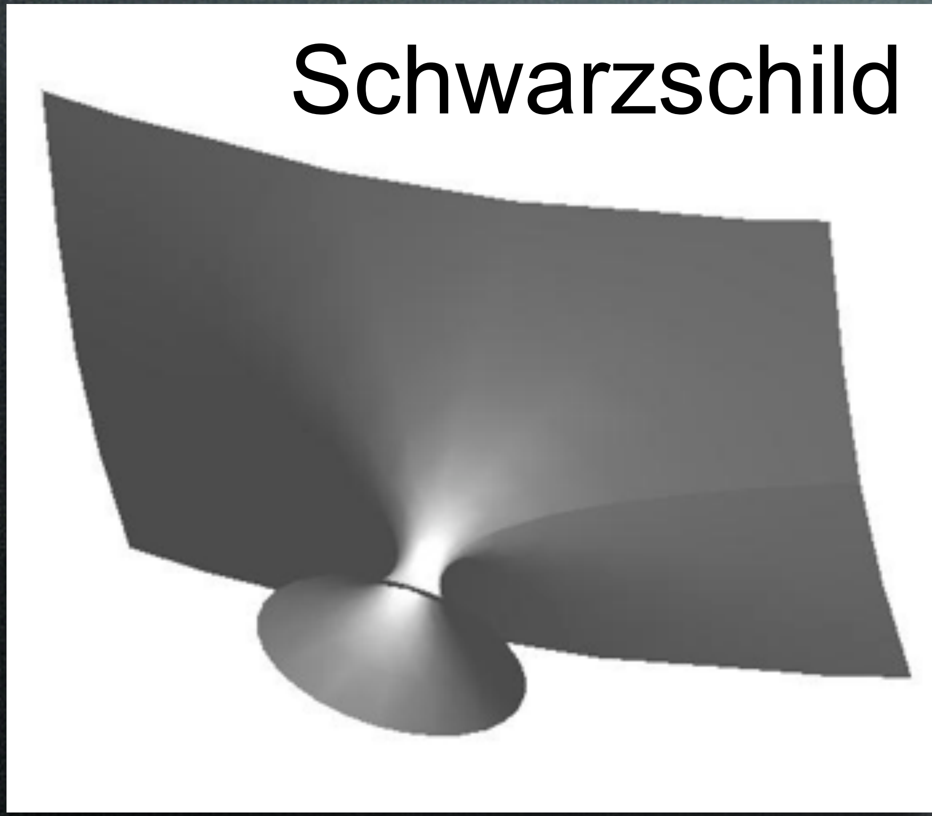

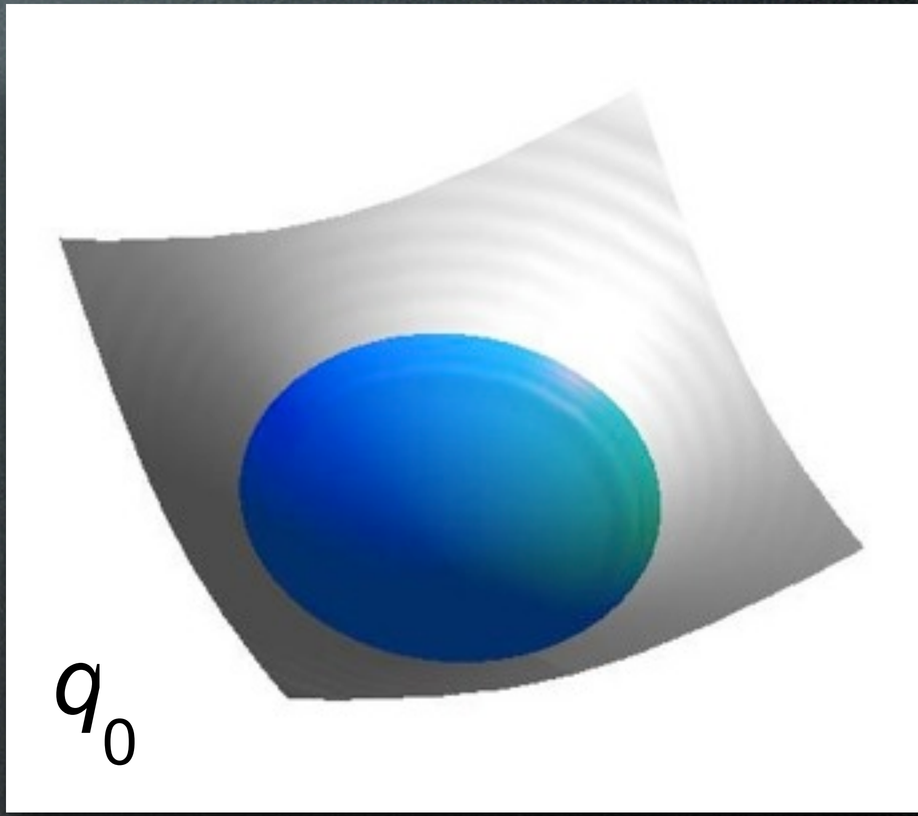

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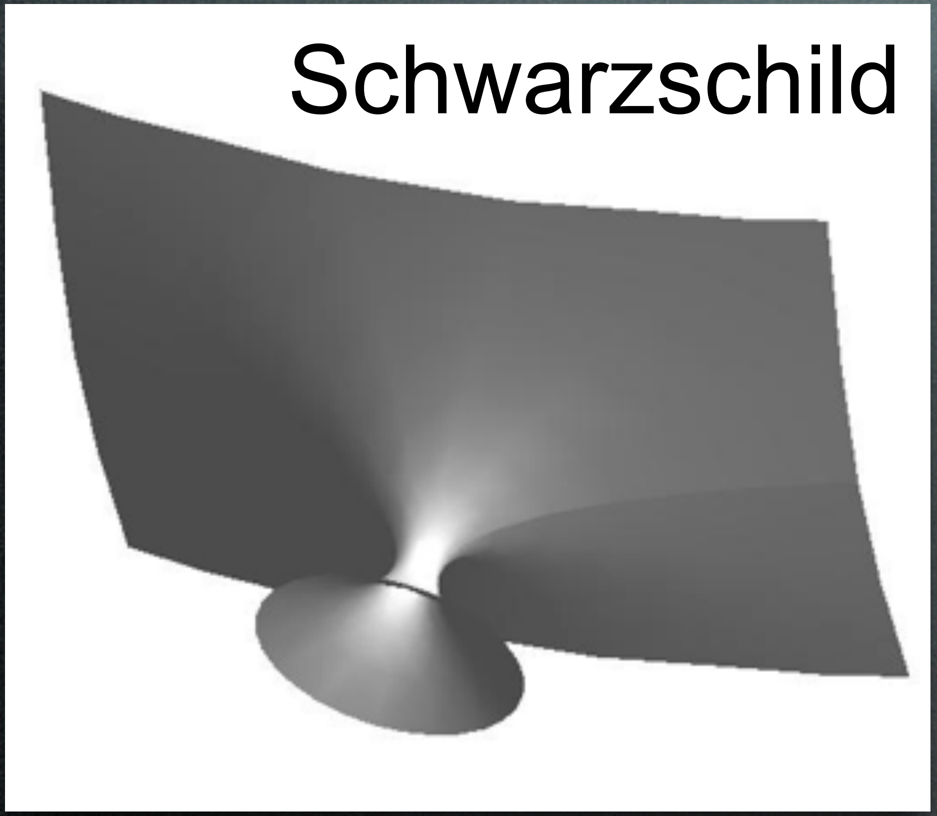

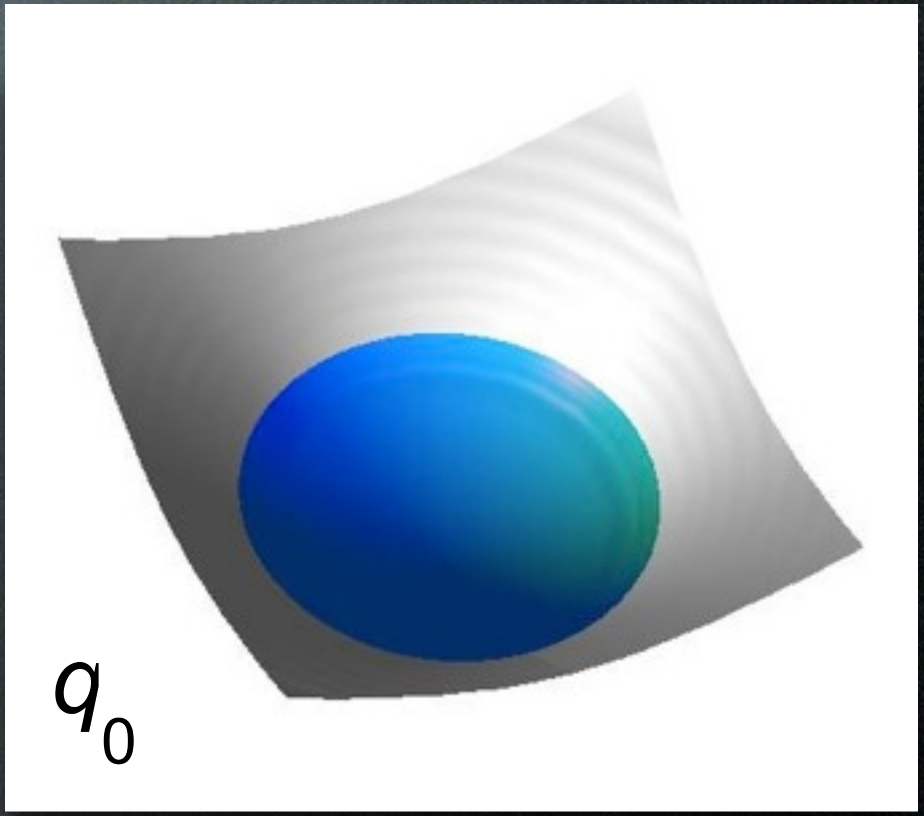

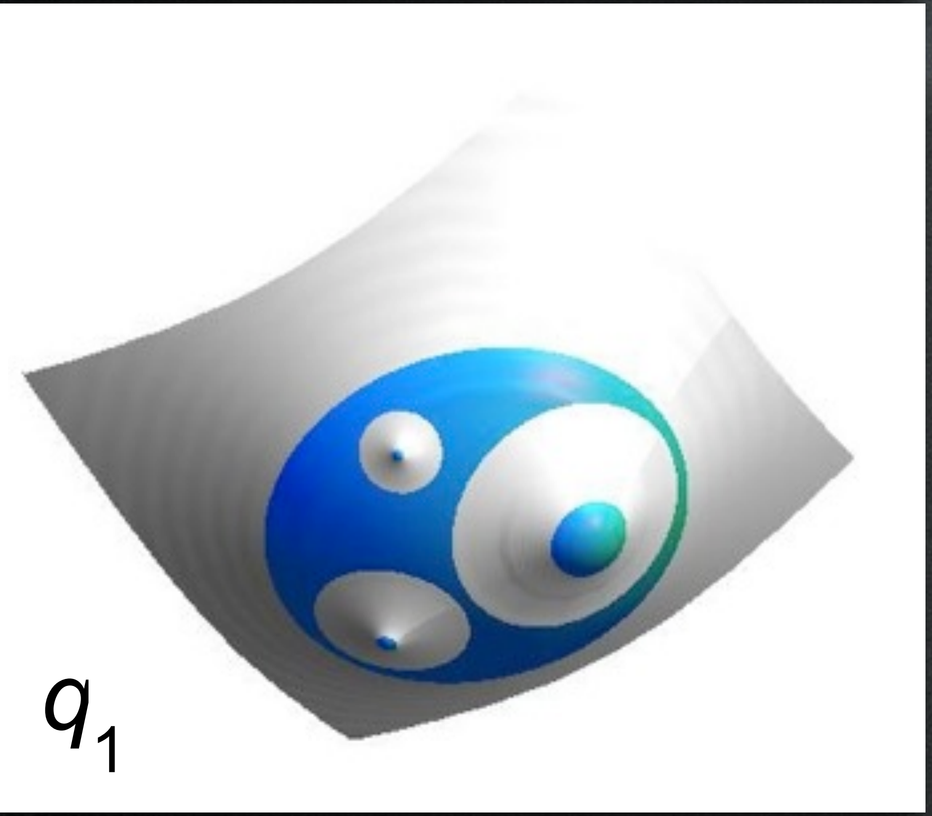

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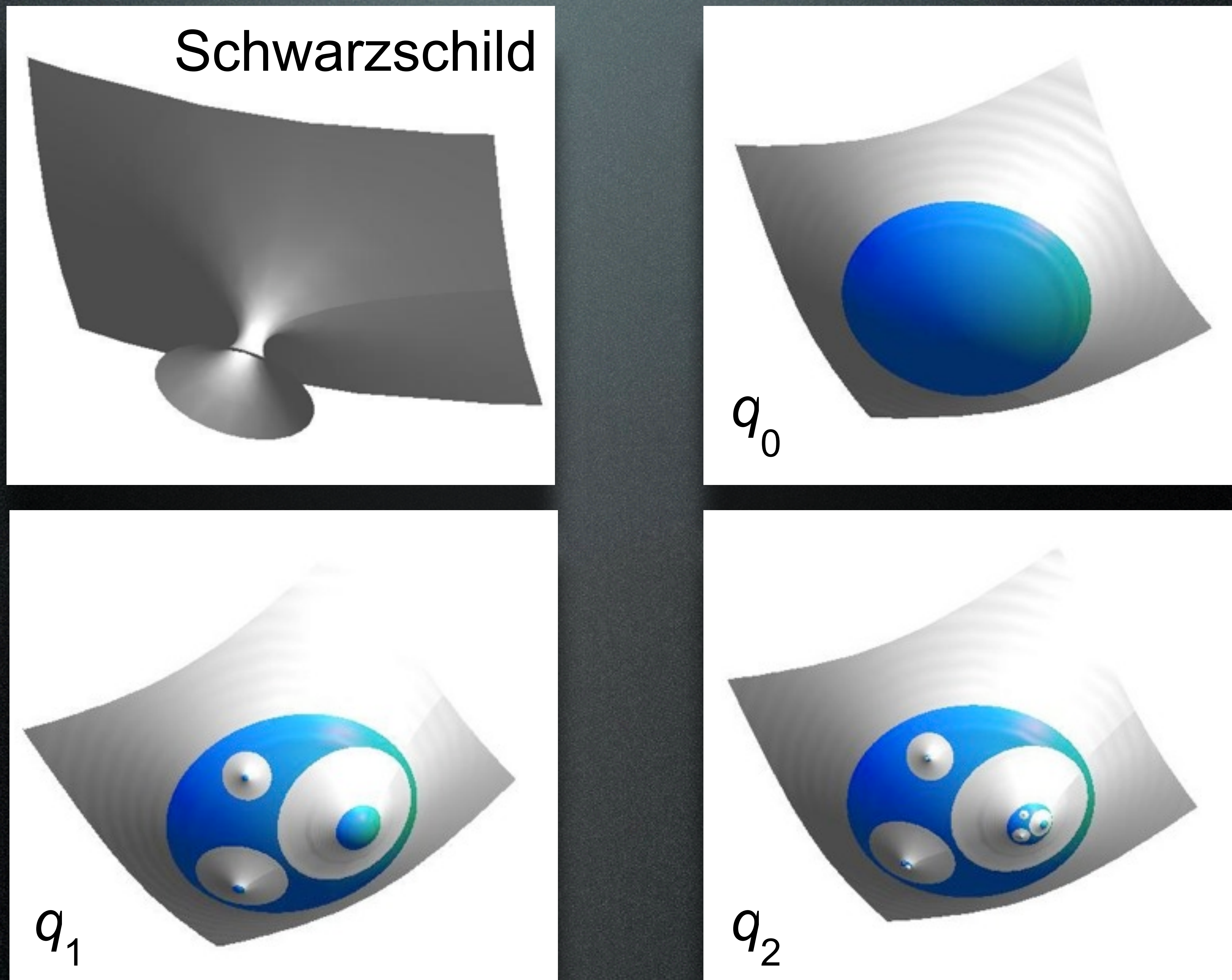


MULTISCALE FOAM SOLUTION

Properties of the configuration

1. Belongs to the swiss-cheese class of solutions (cut and paste)

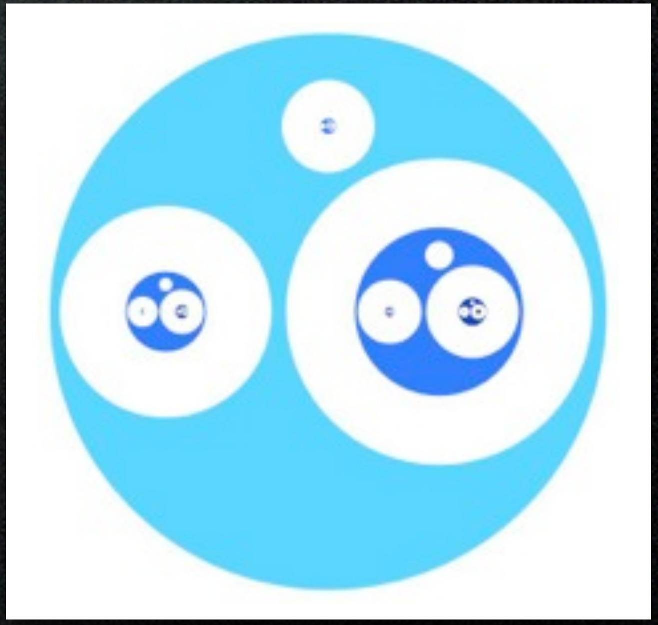
2. Self-similar

3. No pressure ⇒ time evolution can be given in exact form (recollapse)

4. Parametrized by R, ρ, $\alpha_i = V_i / V$, N

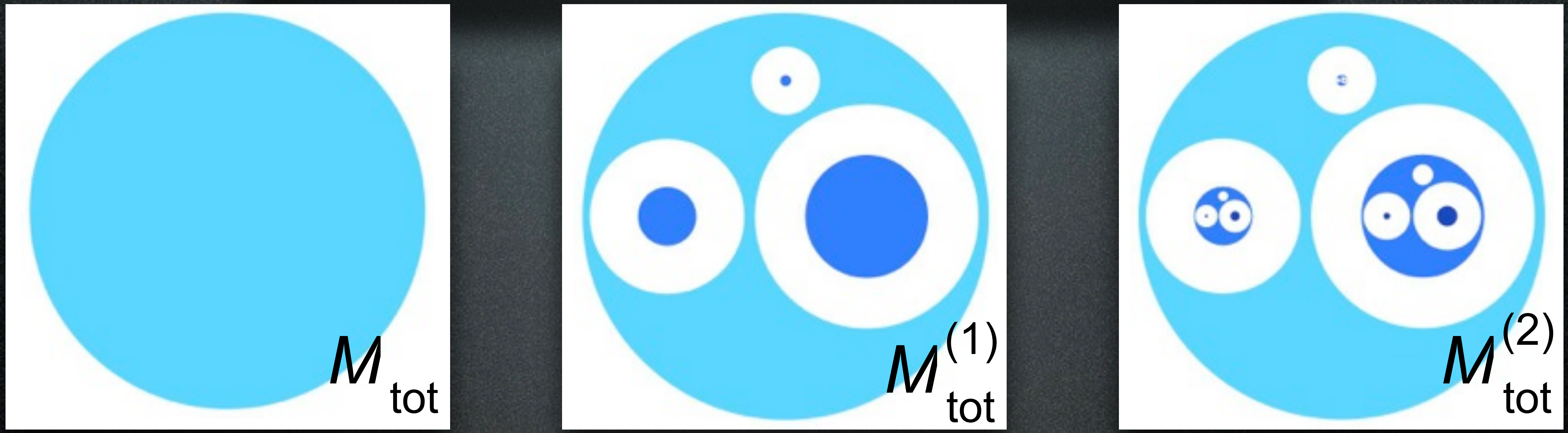
5. $\epsilon = G \rho R^2$ - dimensionless parameter measuring the GR effects, **universal**

6. Outside looks like a spherically symmetric solution with M_{ADM} independent of N



Multiscale foam solution

$$M^{(N)}_{tot} = M_{ADM} + \Delta M + \Delta M^{(1)}_{tot} + \cdots + \Delta M^{(N)}_{tot}$$



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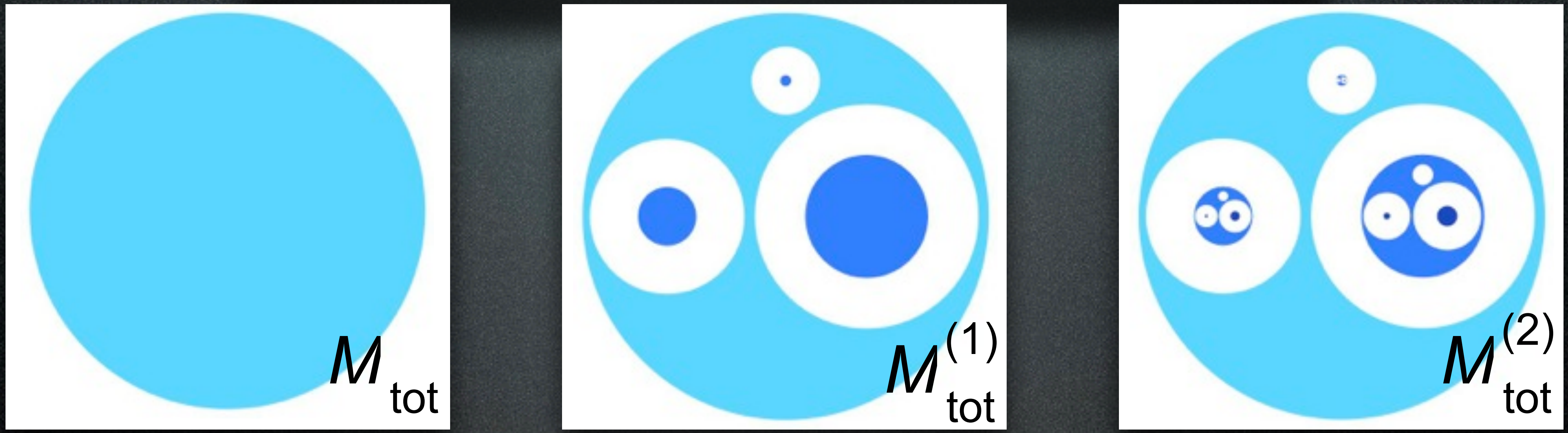

$$\Delta M_{tot}^{(1)} = \delta(R, \rho, \alpha_i) \cdot M_{tot}$$

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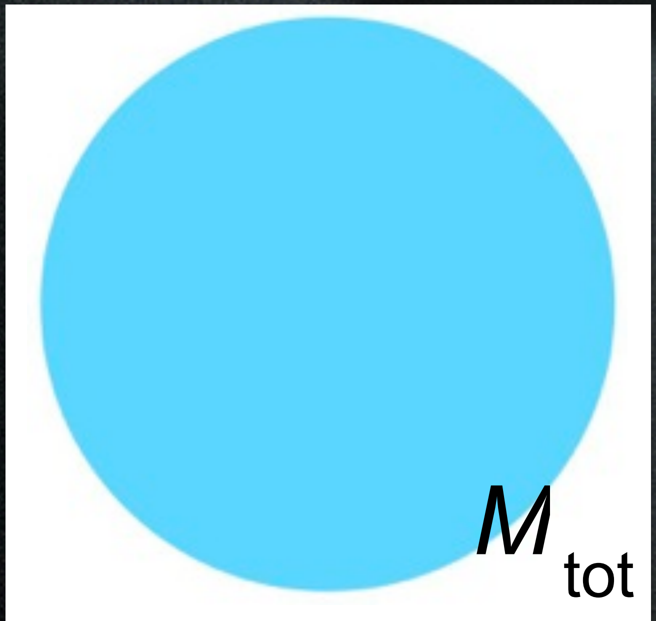

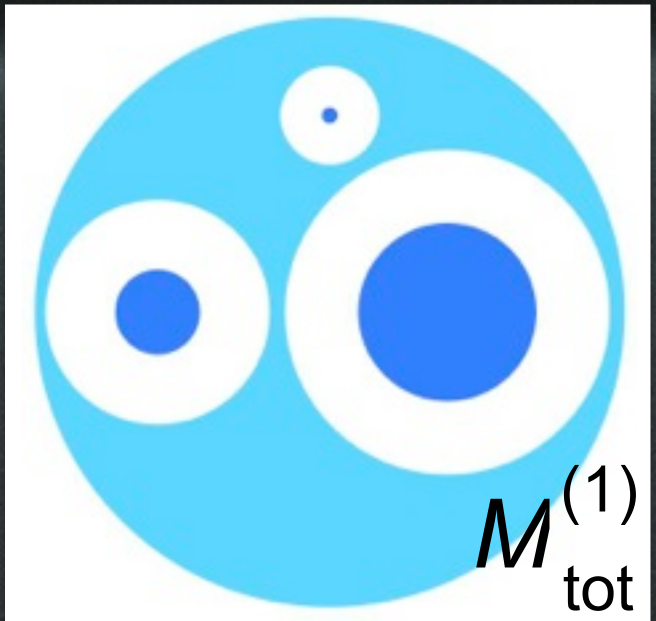

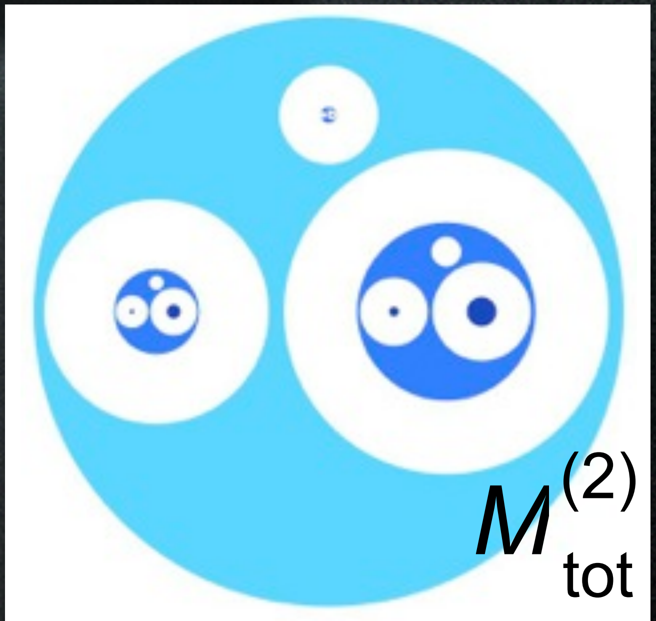

$$\Delta M_{tot}^{(1)} = \delta(R, \rho, \alpha_i) \cdot M_{tot}$$

$$\Delta M_{tot}^{(k)} = \delta \cdot M_{tot} \cdot \Gamma^{k-1}$$

$$\Gamma = \delta + \sum_i \alpha_i$$

MULTISCALE FOAM SOLUTION

$$M_{tot}^{(N)} = M_{ADM} + \Delta M + \Delta M_{tot}^{(1)} + \cdots + \Delta M_{tot}^{(N)}$$

$$M_{tot}^{(N)} = (M_{ADM} + \Delta M)\left(1 + \delta\,\frac{1-\Gamma^N}{1-\Gamma}\right)$$

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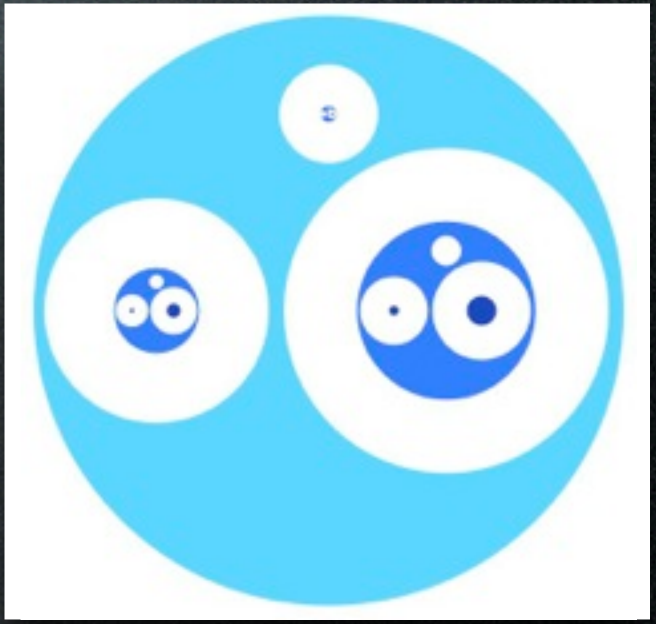

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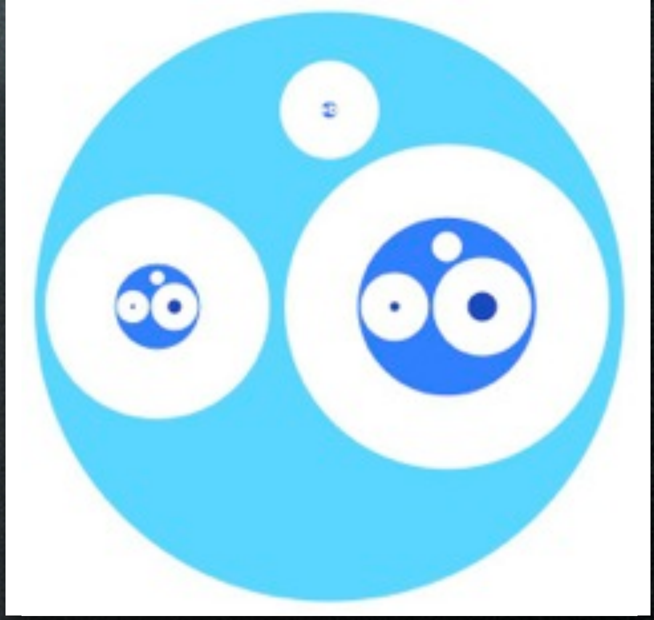

very weakly relativistic configuration (“almost Newtonian”)

$\Rightarrow \Delta M \ll M_{ADM}$, $\delta \ll 1$

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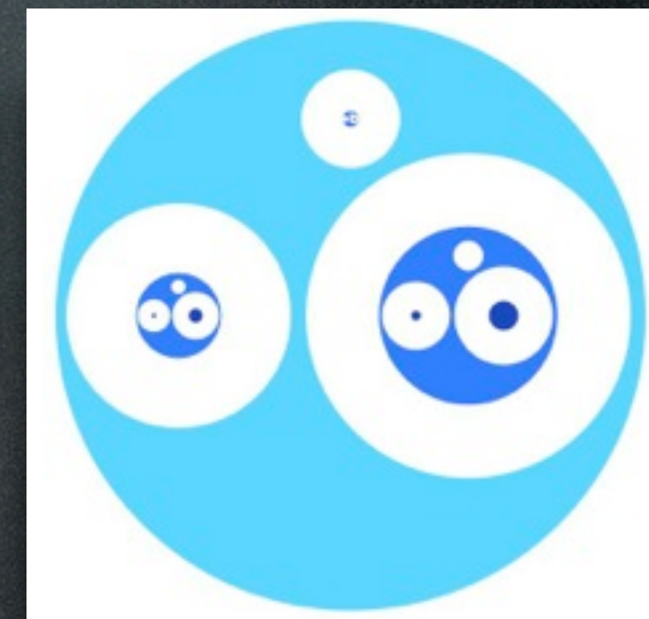

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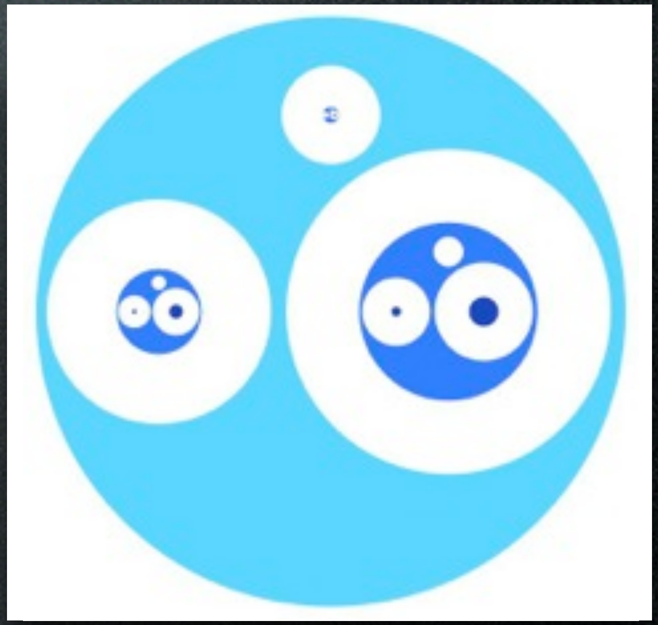

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Multiscale foam solution

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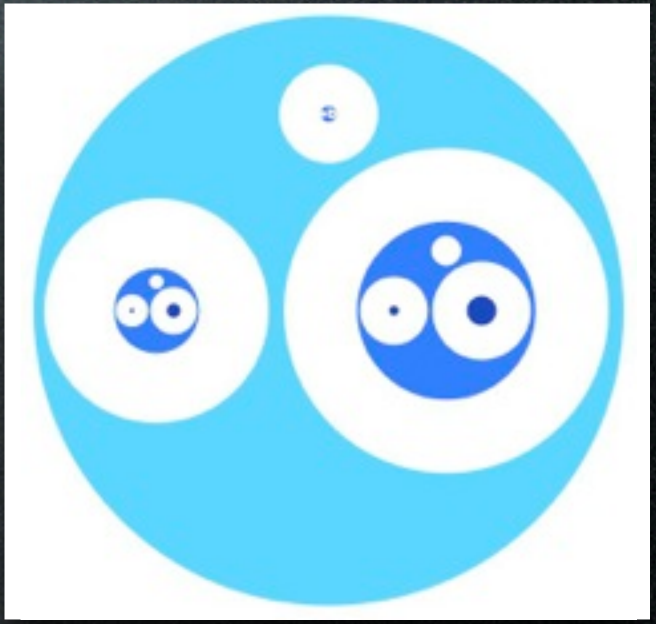

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can be as large as we want

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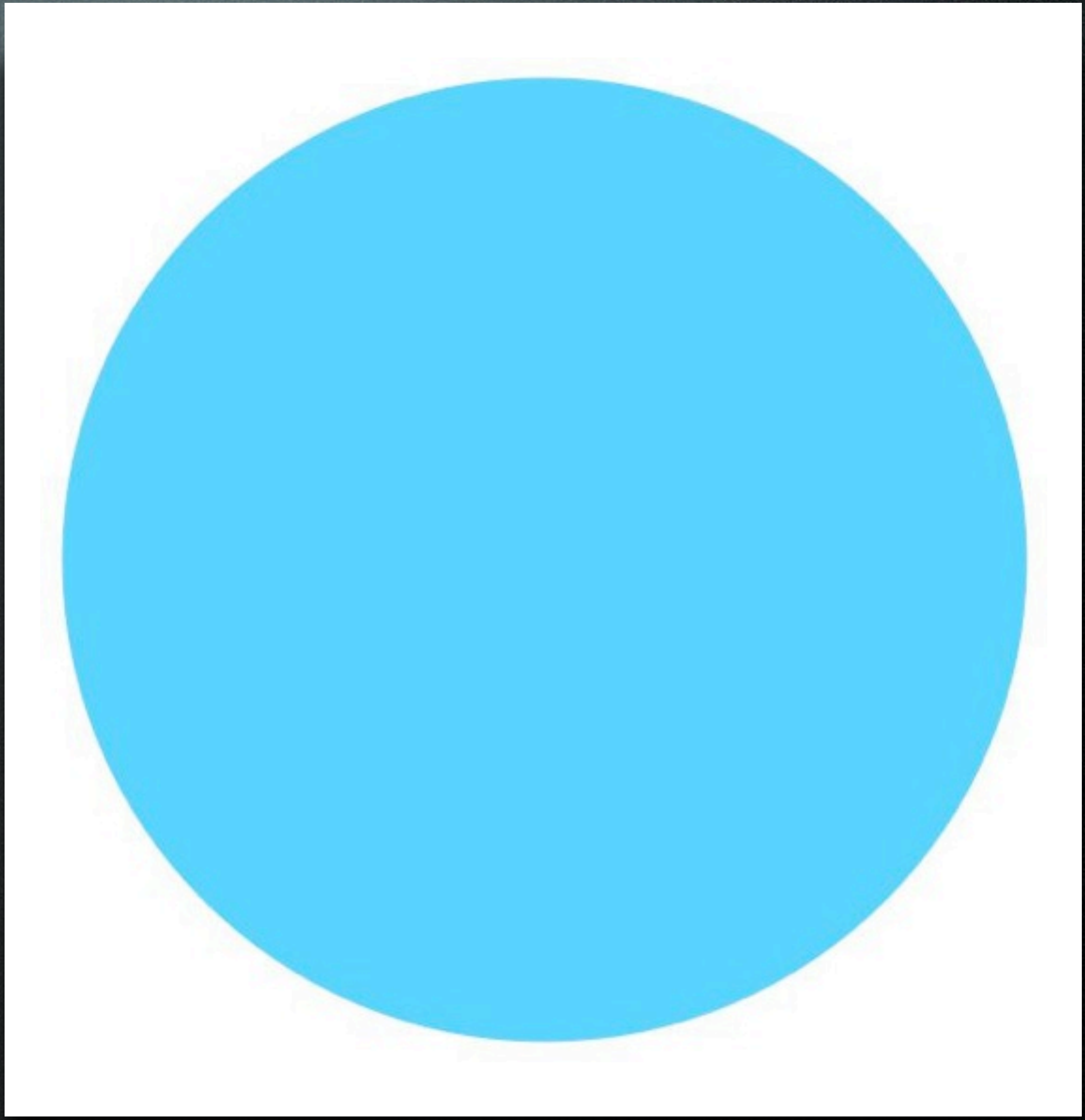
Multiscale foam solution

Making $\sum \alpha_i \simeq 1$



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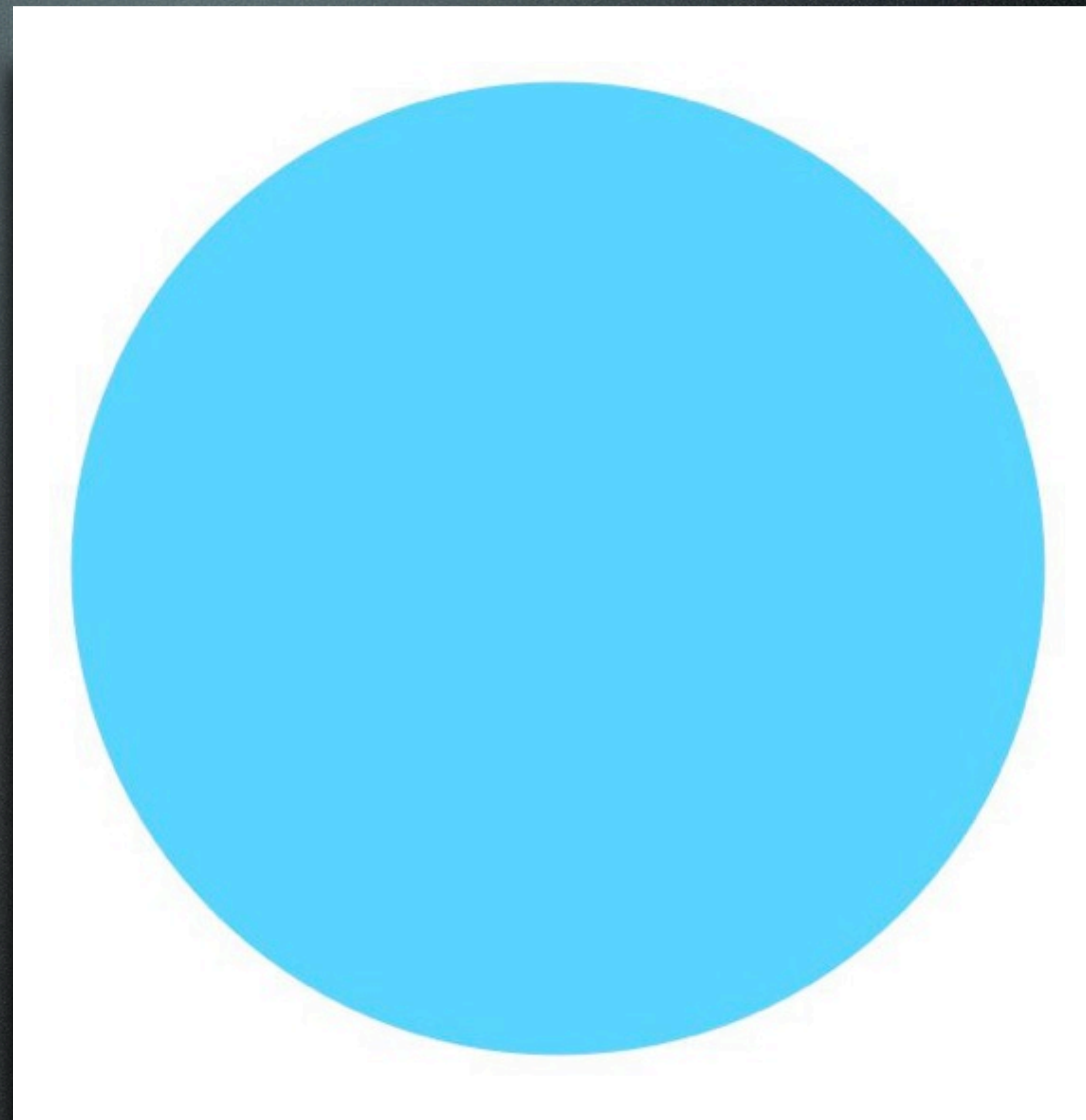
need to “eat up” as large fraction of the original sphere as possible



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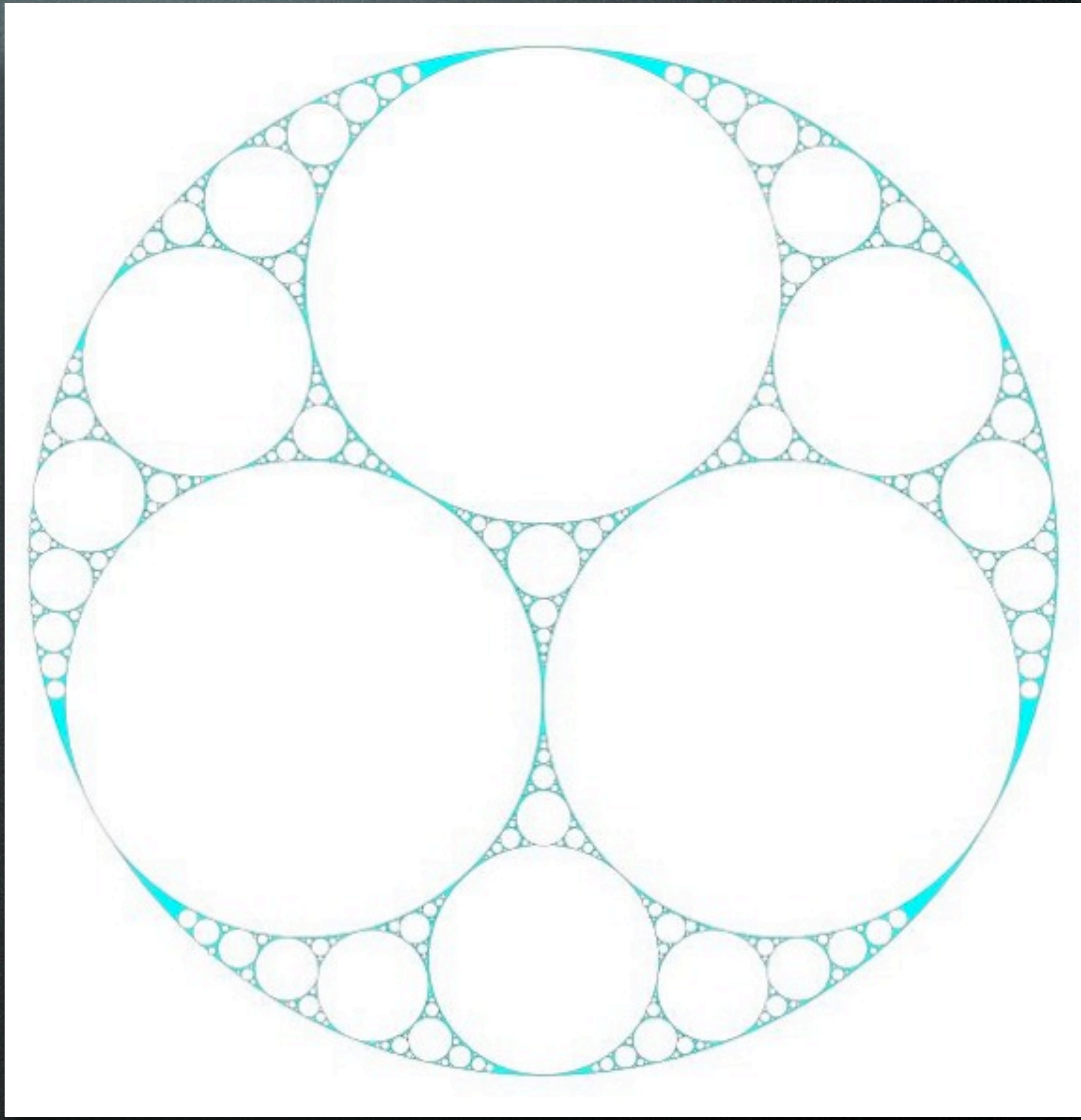

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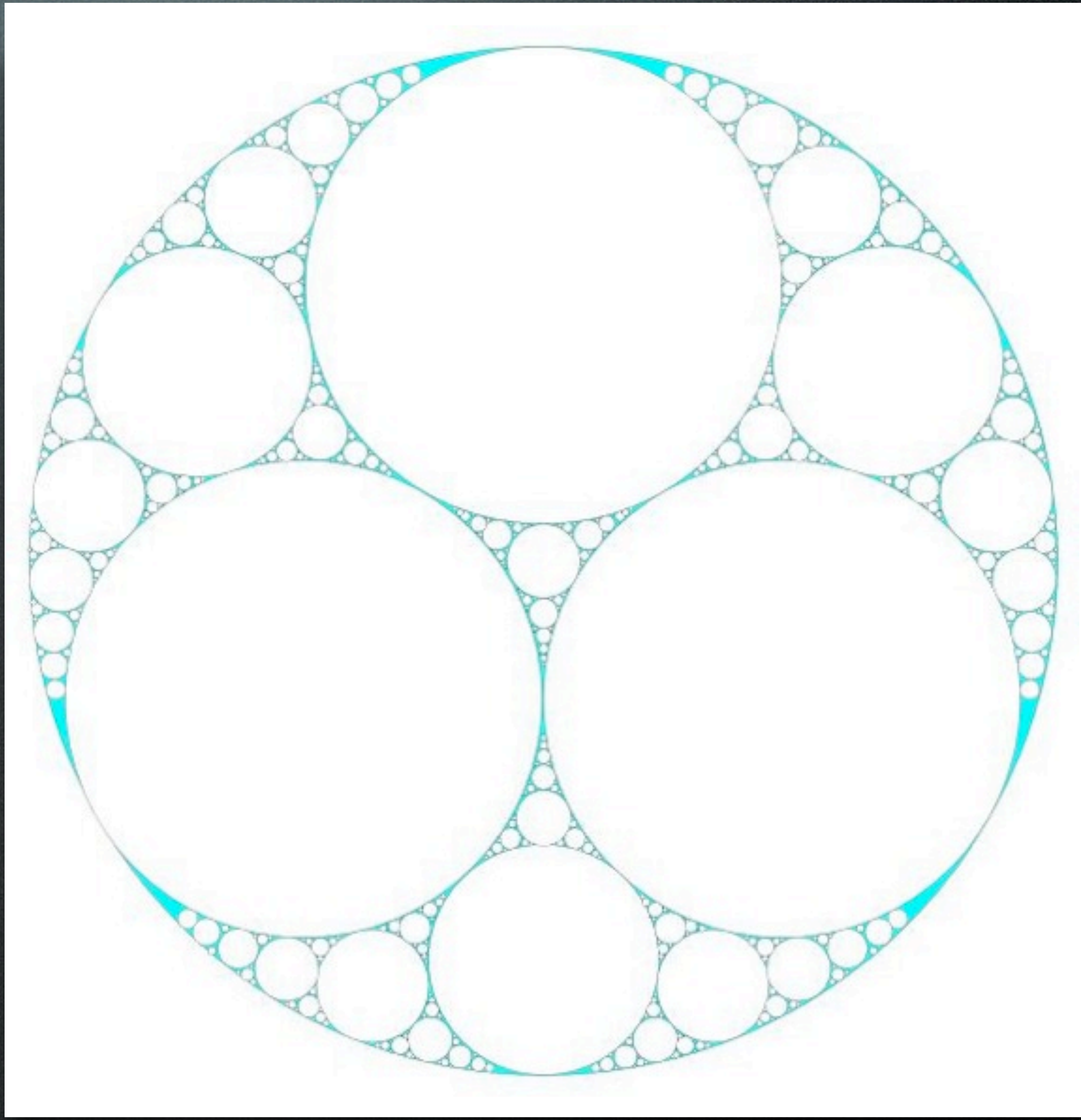
Apollonius of Perga (3rd-2nd century BC)



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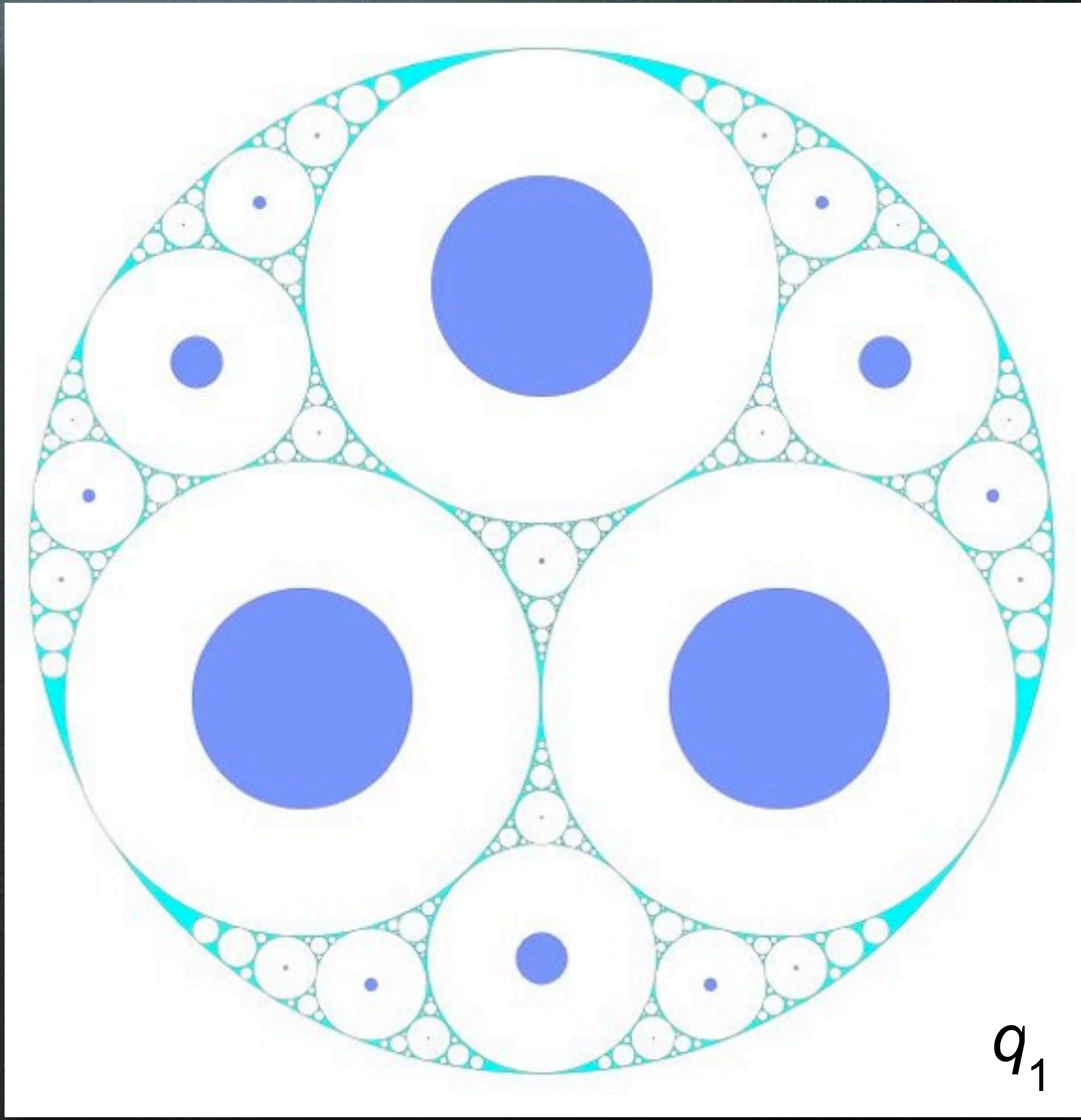

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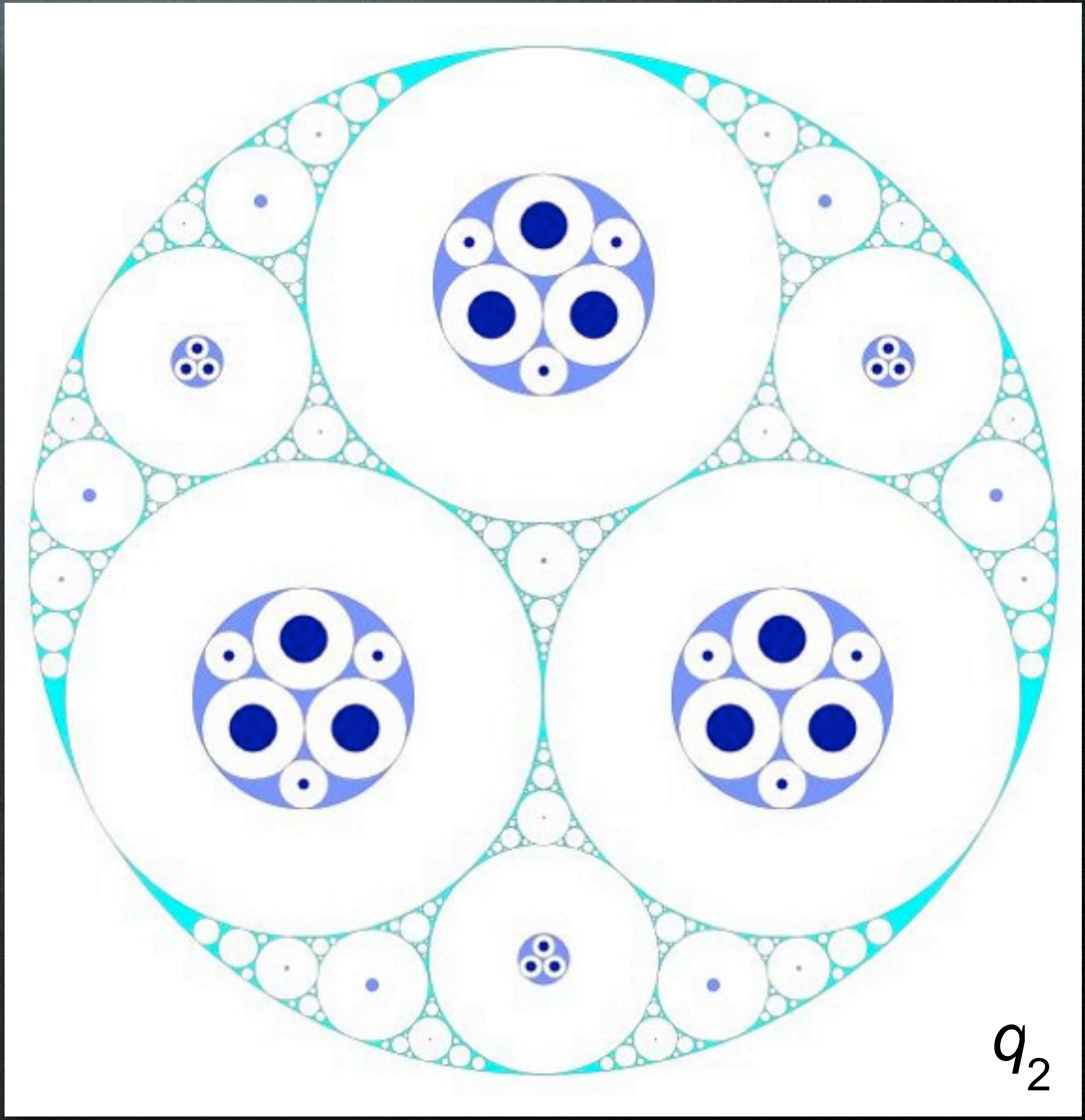
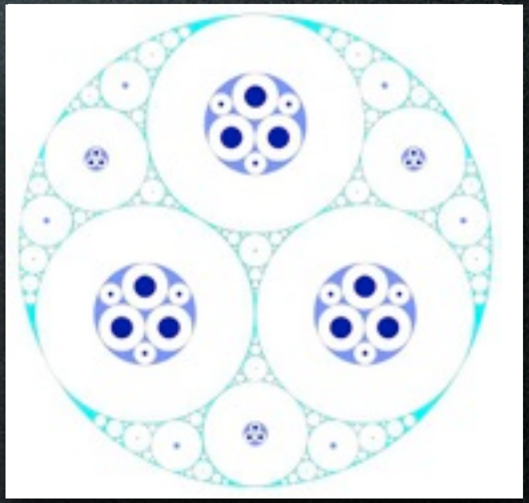


MULTISCALE FOAM SOLUTION

Paradoxical properties



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solution with a complicated structure extending over many scales (voids, overdense regions)

Multiscale foam solution

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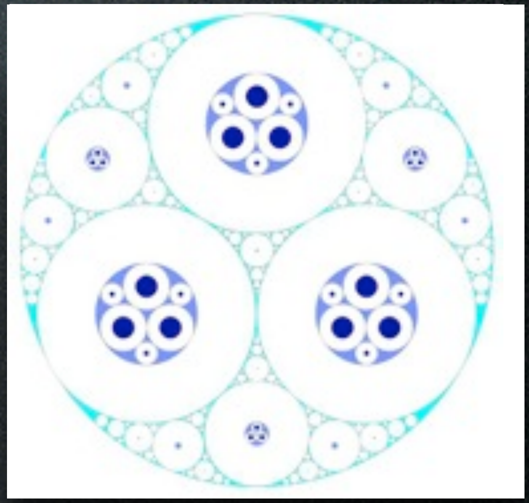

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it seems it should be well described by linearized theory (Newtonian potential, Poisson equation)...

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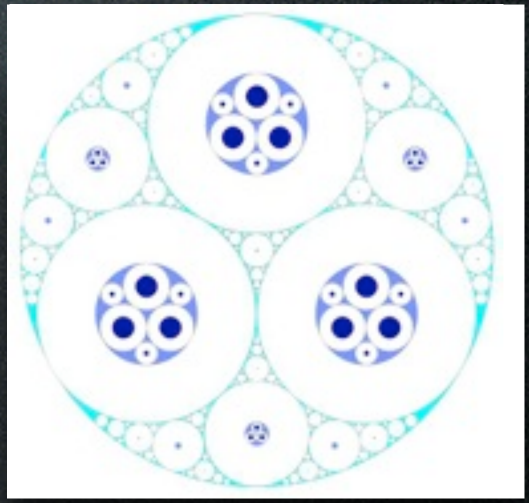

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between the mass of constituents
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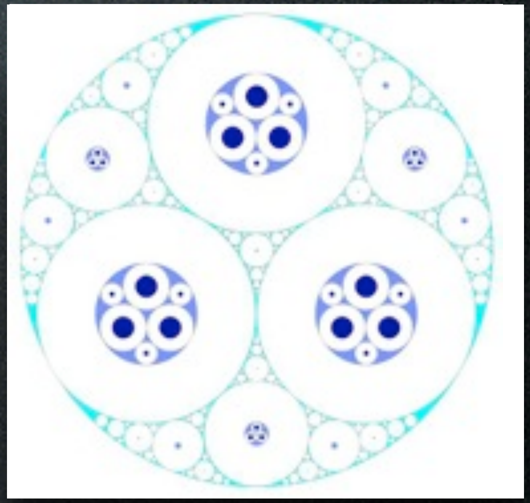

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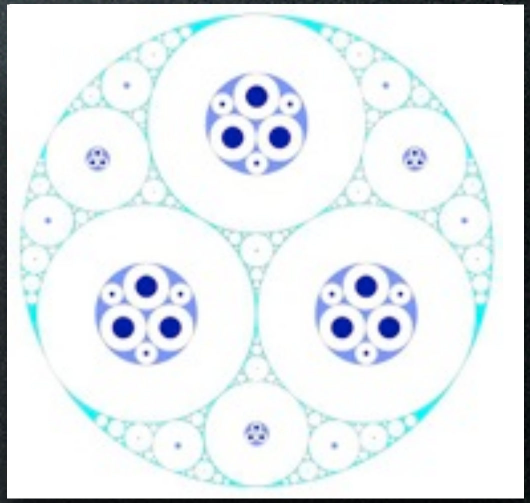

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“naive” linearization fails

COARSE-GRAINING

Explanation

we need to get rid of the structure first to compare

M_{ADM} and $M^{(N)}_{tot}$

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each coarse-graining step q_k to q_{k-1} introduces a tiny correction due to non-linearity of GR

Coarse-graining

Explanation

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M_{ADM} and $M^{(N)}_{tot}$

each coarse-graining step q_k to q_{k-1} introduces a tiny correction due to non-linearity of GR

...but many steps needed (a lot of nested) structure the effect cumulates as we pass over all scales

Dimensionless parameters

$$\frac{M_{tot} - M_{ADM}}{M_{ADM}} = x(\varepsilon)$$

$\epsilon \ll 1$

Dimensionless parameters

$$\frac{M_{tot}^{(N)} - M_{ADM}}{M_{ADM}} = x\left(\varepsilon, D\right)$$

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Dimensionless parameters

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$$D = \log\left(\frac{R_{hom}}{R_{min}}\right)$$

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depth of structure

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homogeneity scale

depth of structure

DIMENSIONLESS PARAMETERS

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smallest ripples scale

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homogeneity scale

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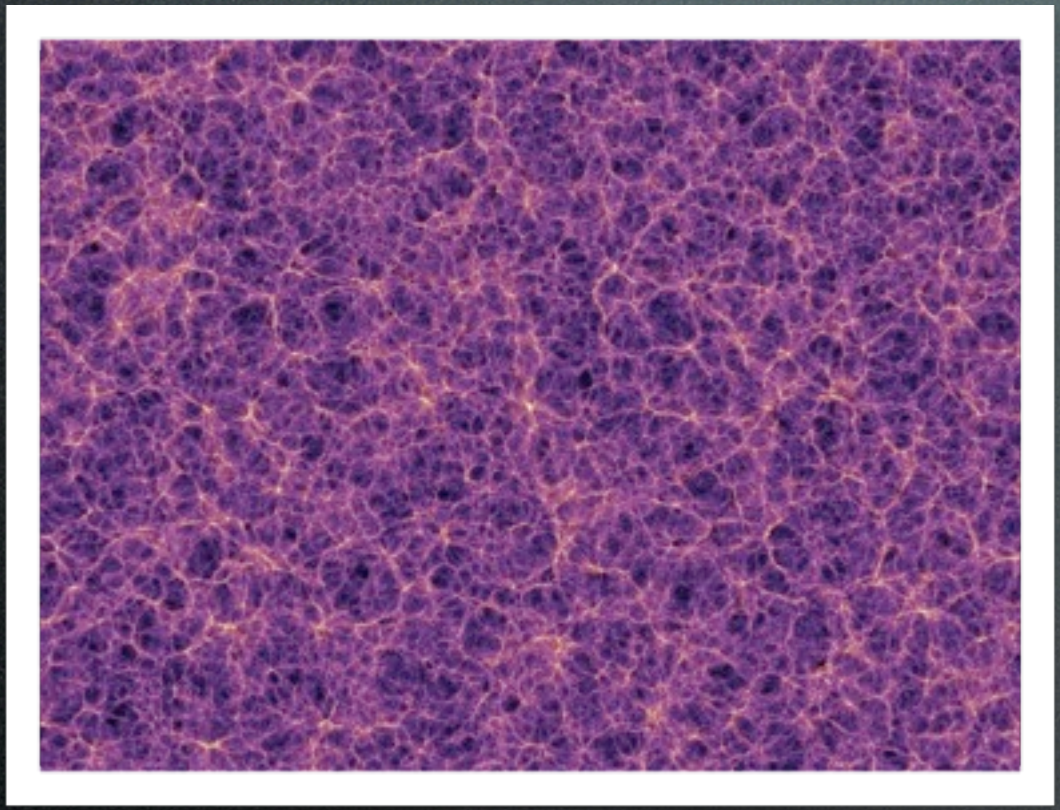
smallest ripples scale

$$x = C \cdot \varepsilon D + O(\varepsilon^2)$$

Remarks

- The object does not have to be exactly self-similar to yield this effect. It suffices if it has **sufficiently deep nested structure**.
- Large, **cumulative** effect of GR nonlinearity despite seemingly Newtonian matter distribution
- Correct approach to the problem: successive, exact **coarse-graining over larger and larger scales** until we hit the homogeneity scale + **renormalization group**
- If no nested structure, the coarse-graining does not produce significant effects outside the strongly relativistic regime (M.K. Classical and Quantum Gravity **31** (2014) 085002*).*
- Nested structure **ubiquitous in cosmology and astrophysics!** Need to check if nonlinearity of GR doesn't play a role here...

NESTED STRUCTURE IN COSMOLOGY



voids, walls, filaments,
large quasar groups,
galaxy clusters...

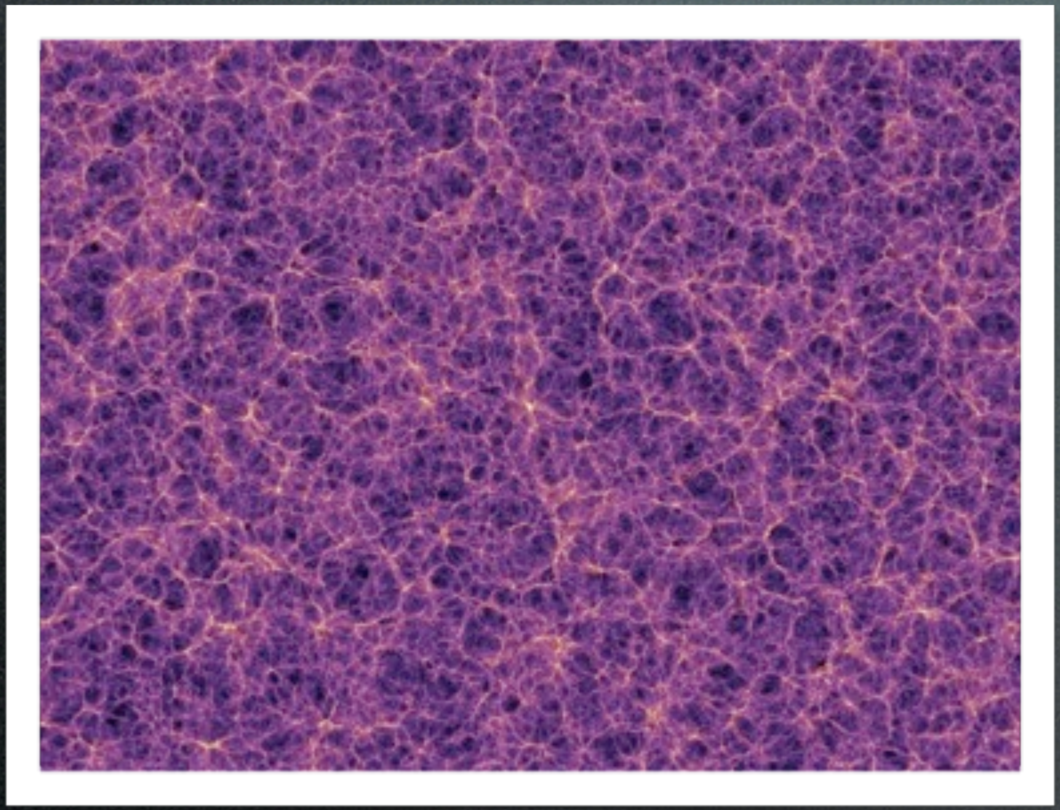
Huge-LQG

$$L_{hom} = 1.2 \cdot 10^9 \text{pc}$$

distance between stars in the Galaxy

$$L_{stars} \approx 1 \text{pc}$$

Nested structure in cosmology



voids, walls, filaments,
large quasar groups,
galaxy clusters...

Huge-LQG

$$L_{hom} = 1.2 \cdot 10^9 \text{pc}$$

$$D \approx \log 10^9 \approx 20$$

distance between stars
in the Galaxy

$$L_{stars} \approx 1\text{pc}$$

Ishibashi and Wald's argument

Class. Quantum Grav. **23** 235-250 (2006)

Can the Acceleration of Our Universe Be Explained by the Effects of Inhomogeneities?

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February 4, 2008

Ishibashi and Wald's argument

Class. Quantum Grav. **23** 235-250 (2006)

Can the Acceleration of Our Universe Be Explained by the Effects of Inhomogeneities?

Abstract

No. It is simply not plausible that cosmic acceleration could arise within the context of general relativity from a back-reaction effect of inhomogeneities in our universe, without the presence of a cosmological constant or "dark energy." We point out that our universe appears to be described very accurately on all scales by a Newtonianly perturbed FLRW metric. (This assertion is entirely consistent with the fact that we commonly encounter $\delta\rho/\rho > 10^{30}$.) If the universe is accurately described by a Newtonianly perturbed FLRW metric, then the back-reaction of inhomogeneities on the dynamics of the universe is negligible. If not, then it is the burden of an alternative model to account for the observed properties of our universe. We emphasize with concrete examples that it is

Summary

- An example of an object made of dust, containing deep, nested structure
- voids and overdense regions, but non-relativistic (mass small compared to their size)
- nevertheless large net effect of the nonlinearity of GR (mass deficit) due to very deep structure (cumulative effect)
- Questions arising: do we need to take into account the effects of GR and coarse-graining in cosmology or astrophysics of galaxy clusters? (linear perturbations may not be enough, even though the structure seems to be possible to be described by linearized GR at all scales!)
- Coarse-graining formalism in GR needed - work in progress