Towards solving generic cosmological singularity problem

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Evidence for the existence of the cosmological singularity

- **observational cosmology:**
  the Universe has been expanding for almost 14 billion years
  (emerged from a state with extremely high energy densities
  of physical fields)

- **theoretical cosmology:**
  almost all known general relativity models of the Universe:
  (Lemaître, Kasner, AdS, Friedmann, Bianchi, ..., BKL)
  predict the existence of cosmological singularities

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means that this classical theory is incomplete

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Some intriguing questions concerning the quantum phase of the Universe:

- What is the energy scale?
- What is the mechanism of the transition: quantum phase $\Leftrightarrow$ classical phase?
- How to relate theory with cosmic observations?
  - What is the origin of inflation?
  - What is the origin of tiny fluctuations visible in CMB?
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- **Canonical** quantization based on the **Holst** action and loop geometry
  - Dirac’s LQC\(^1\) := ‘first quantize then impose constraints’
  - RPS LQC\(^2\) := ‘first solve constraints then quantize’
- **Coherent states**\(^3\) and **canonical**\(^4\) quantizations based on the Hilbert-Einstein action

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Quantum FRW model: summary of results obtained within RPS LQC approach

- Cosmic singularity problem of FRW model can be resolved by using the loop geometry: big bang turns into big bounce.

- Discreteness of the spectra of the volume operators may favor a foamy structure of space at short distances: no dispersion of cosmic photons$^5$ up to the energy $5 \times 10^{17}$ GeV.


- Evolution of a quantum phase can be described in terms of the self-adjoint true Hamiltonian:
  - expectation values of quantum variables coincide with corresponding classical variables.
  - Heisenberg’s uncertainty relation is perfectly satisfied during the entire evolution of the universe.

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Main papers:

[1] P. Dzierżak, P. Małkiewicz and W. P.,
‘Turning big bang into big bounce: I. Classical dynamics’,

‘Turning big bang into big bounce: II. Quantum dynamics’.

‘Energy scale of the Big Bounce’,

‘Observables for FRW model with cosmological constant’.

‘Evolution in bouncing quantum cosmology’,

‘Gaussian state for the bouncing quantum cosmology’,

‘Quantum states of the bouncing universe’,
Challenge in Cosmology:


- FRW metric is dynamically **unstable** in the evolution towards the singularity (breaking of isotropy).
- Bianchi type metric is dynamically **unstable** in the evolution towards the singularity (breaking of homogeneity).
- BKL scenario is thought to be **generic** solution to GR near CS
  - does not rely on **any symmetry** conditions;
  - corresponds to **non-zero** measure subset of all initial conditions;
  - solution is **stable** against perturbation of initial conditions.
- BKL appears in the low energy limit of **superstring** models.
- **Application** of non-singular **quantum** BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.
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Quantization of the Bianchi IX model:

- **Dynamics of Bianchi-IX is the best prototype for the BKL scenario**\(^7\)

- **Questions to answer:**
  - What happens to the oscillatory/chaotic dynamics after the imposition of quantum rules onto the dynamics?
  - What happens to the classical singularity of the Bianchi-IX at the quantum level?
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The world-interval of spacetime

Presented results are based on the forthcoming paper

- The line element of the homogeneous Bianchi type model in spacetime $\mathcal{M} \mapsto \Sigma \times \mathbb{R}$, where $\Sigma$ is spacelike

$$ s^2 = -\mathcal{N}(t) \ t^2 + \sum_i q_i(t) \omega^i \otimes \omega^i, $$

where $\omega^i$ are 1-forms on $\Sigma$ invariant with respect to the action of a simply transitive group of motions on the leaf and subject to

$$ \omega^i = \frac{1}{2} C^i_{jk} \omega^j \wedge \omega^k, $$

where $C^i_{jk}$ are structure constants of the corresponding Lie algebra.
Classical Hamiltonian

In the Misner type parametrization of phase space we have

\[ \mathcal{H} = \mathcal{N}(t) \left( \frac{2\pi G}{3c^2a^3} \left( a^2 p_a^2 - p_+^2 - p_-^2 \right) - \frac{c^4}{32\pi G} a W_n(\beta_{\pm}) \right) \approx 0, \quad (3) \]

where \((a, \beta_{\pm}; p_a, p_{\pm})\) are canonical variables of the kinematical phase space.

Well known homogeneous models can be obtained as follows:

- FRW, by taking \(W_n(\beta_{\pm}) = 0\) and \(p_{\pm} = 0\);
- Bianchi-I, corresponds to \(W_n(\beta_{\pm}) = 0\);
- Bianchi-II, has \(W_n(\beta_{\pm}) = n^2 e^{8\beta_{+}}\) and \(n > 0\).
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The Bianchi IX model is defined by

\[ W_n(\beta_\pm) = n^2 e^{-4\beta_+} \left( \left( e^{6\beta_+} - 2 \cosh(2\sqrt{3}\beta_-) \right)^2 - 4 \right), \quad n > 0. \]  

The potential \( W_n \) is bounded from below and reaches its (absolute) minimal value at \( \beta_\pm = 0 \), with \( W_n(0) = -3n^2 \). \( W_n \) has \( C_{3v} \) symmetry and is asymptotically confined except for three directions:

(i) \( \beta_- = 0, \beta_+ \to -\infty \),
(ii) \( \beta_+ = \beta_- / \sqrt{3}, \beta_- \to +\infty \),
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The Bianchi IX model is defined by

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Redefining the phase space variables, to highlight possible approximation, by introducing the canonical pair $(q = a^{3/2}, p = 2p_a/(3\sqrt{a}))$ we get

$$\mathcal{H} = \mathcal{N}(t) \left( \frac{2\pi G}{3c^2} \left( \frac{9}{4} p^2 - \frac{p_+^2 + p_-^2}{q^2} \right) - \frac{c^4}{32\pi G} q^{2/3} W_n(\beta_{\pm}) \right). \quad (5)$$

It results from Eq. (5) that near the singularity, $q = 0$, we may treat $q$ as heavy degree of freedom, and $\beta_{\pm}$ as light degrees of freedom. It is so because ‘the mass’ of the $\beta_{\pm}$ behaves as $q^2$, while ‘the mass’ of $q$ is fixed.

Therefore, we may quantize our system by using the adiabatic approximation.
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Classical Hamiltonian (cont)

For the purpose of the adiabatic quantization:

\[ \mathcal{H} = N(t) \left( \frac{3\pi G}{2c^2} p^2 - \mathcal{H}_\pm \right), \]  

(6)

where

\[ \mathcal{H}_\pm := \frac{2\pi G}{3c^2 q^2} (p^2_+ + p^2_-) + \frac{c^4}{32\pi G} q^{2/3} W_n(\beta_\pm). \]  

(7)

- \( \beta_\pm = 0 = p_\pm \) corresponds to the classical ground state of the anisotropy Hamiltonian \( \mathcal{H}_\pm \). Thus, FRW may be treated as a special case of Bianchi-IX, where the anisotropy degrees of freedom are frozen in their (classical) ground state.

- We cannot quantize the FRW model alone, because we should take into account the effect of quantum ‘zero point energy’ generated by the quantized anisotropy degrees of freedom of the Bianchi-IX model.
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In what follows we apply the modified Dirac quantization method:

- quantizing $\mathcal{H}$ (all degrees of freedom) and get $\hat{\mathcal{H}}$,
- finding semi-classical expression $\hat{\mathcal{H}}$ of $\hat{\mathcal{H}}$,
- implementing constraint $\hat{\mathcal{H}} = 0$,
- making adiabatic approximation.

Since $(q, p) \in \mathbb{R}_+ \times \mathbb{R}$ and $(\beta_\pm, p_\pm) \in \mathbb{R}^4$:

- we apply affine coherent states quantization to $(q, p)$,
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Quantum Hamiltonian

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The quantum Hamiltonian $\hat{\mathcal{H}}$ reads

$$\hat{\mathcal{H}} = \mathcal{N}(t) \left( 3\pi G \left( \frac{\hat{p}^2 + \hbar^2 \mathcal{K}_1}{\hat{q}^2} \right) - \frac{2\pi G}{3c^2} \mathcal{K}_2 \frac{\hat{p}^2 + \hat{q}^2}{\hat{q}^2} - \frac{c^4}{32\pi G} \mathcal{K}_3 \hat{q}^{2/3} W_n(\hat{\beta}_\pm) \right),$$

where the $\mathcal{K}_i$:

$$\mathcal{K}_1 = \frac{1}{4} \left( 1 + \frac{K_0(\nu)}{K_1(\nu)} \right), \quad \mathcal{K}_2 = \left( \frac{K_2(\nu)}{K_1(\nu)} \right)^2, \quad \mathcal{K}_3 = \frac{K_{5/3}(\nu)}{K_1(\nu)^{1/3} K_2(\nu)^{2/3}},$$

and where the $K_\alpha(\nu)$ are modified Bessel functions.

$\hat{p}_\pm = -i\hbar \partial_\beta_\pm$, and $\hat{\beta}_\pm$ defined as $\beta_\pm$, acting on $L^2(\mathbb{R}^2, d\beta_+ d\beta_-)$; $\hat{p} = -i\hbar \partial_q$, and $\hat{q}$ defined as $q$, acting on $L^2(\mathbb{R}_+, dq)$. 

$\mathcal{N}(t)$
Semi-classical approximation

Near the singularity we have

\[ \hat{H}_\pm(q) = \frac{2\pi G}{3c^2} \mathcal{K}_2 \frac{\hat{p}_+^2 + \hat{p}_-^2}{q^2} + \frac{c^4}{32\pi G} \mathcal{K}_3 q^{2/3} W_n(\beta_\pm). \]  

(10)

Due to the harmonic behavior of \( W_n \) near its minimum:

\[ W_n(\beta_\pm) \approx -3n^2 + 24n^2(\beta_+^2 + \beta_-^2) + o(\beta_\pm^2), \]  

(11)

the harmonic approximation to the eigen energies \( E^{(N)}_\pm(q) \) is possible \((N = 0, 1, \ldots)\):

\[ E^{(N)}_\pm(q) \approx -\frac{3c^4}{32\pi G} \mathcal{K}_3 q^{2/3} n^2 + \frac{\hbar c}{q^{2/3}} n \sqrt{2\mathcal{K}_2 \mathcal{K}_3} (N + 1). \]  

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The Hamiltonian $\hat{H}$ is now replaced by the **averaged** one $\hat{H}_{av}$:

$$\hat{H}_{av} = \mathcal{N}(t) \left( \frac{3\pi G}{2c^2} \left( \hat{p}^2 + \frac{\hbar^2 K_1}{\hat{q}^2} \right) - E^{(N)}_{\pm}(\hat{q}) \right).$$  \hfill (13)

The semi-classical expressions with affine CS, is defined as

$$\hat{\mathcal{H}}_{av}(q, p) = \langle \lambda q, p|\hat{H}_{av}|\lambda q, p \rangle,$$  \hfill (14)

where $\lambda := K_0(\nu)/K_2(\nu)$ is chosen to get $\langle \lambda q, p|\hat{q}|\lambda q, p \rangle = q$ and $\langle \lambda q, p|\hat{p}|\lambda q, p \rangle = p$.

Finally, we obtain

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Imposition of Hamiltonian constraint:

The constraint \( \mathcal{H}_{av}(q, p) = 0 \) reads

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\frac{\ddot{a}^2}{a^2} + k \frac{c^2}{a^2} + s_P^2 c^2 \frac{\mathcal{R}_4}{a^6} = \frac{8\pi G}{3c^2} \rho(a),
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s_P := 2\pi G \hbar c^{-3}, \quad k := \frac{\mathcal{R}_5 n^2}{4}, \quad \rho(a) := \hbar c(N + 1) \frac{n \mathcal{R}_6}{a^4}.
\]

The main features of this quantum corrected model:

- **anisotropy** degrees of freedom produce the ‘radiation-like’ energy density \( \rho(a) \); primordial GW?
- curvature-like term \( k c^2 a^{-2} \), modifies the usual FRW result;
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Resolution of singularity

Equation (16) can be rewritten as

\[ kc^2 + s_P^2 c^2 \frac{\kappa_4}{a^4} - \frac{8\pi G}{3c^2} a^2 \rho(a) \leq 0. \]  

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which defines allowed values of \( a \in [a_-, a_+] \).

The semi-classical trajectories are bounded from below

\[ \frac{s_P}{a_+^2} = \frac{2\kappa_6}{3\kappa_4} n(N + 1) \left( 1 + \sqrt{1 - \frac{f(\nu)}{(N + 1)^2}} \right) \]  

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Periodicity of trajectories

The semi-classical trajectories are periodic. The oscillatory period $T$ of the universe is

$$T = \frac{2t_P}{\sqrt{R_4}} (x_- x_+)^{-3/4} \left( \frac{x_+}{x_-} \right)^{-1/4} E \left( 1 - \frac{x_+}{x_-} \right),$$

(21)

where $t_P = \sqrt{s_P}/c$ is the Planck time, $x_\pm = s_P/a_\pm^2$, and $E$ is the complete elliptic integral of the second kind.
Conclusions

Applying

- **mixed** procedure of quantization (CS+canonical),
- **adiabatic** approximation to the quantized Hamiltonian $\hat{H}$,
- constraint $H = 0$ at the semi-classical level,

it is possible to develop a quantum version of the classical Bianchi-IX model that looks like a modified FRW model.

The main features of this quantum model are:

- the transformation of the quantum energy due to anisotropy degrees of freedom into ‘primordial GW’,
- a curvature-like term $kc^2 a^{-2}$ (already present in the classical Bianchi-IX treatment),
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it is possible to develop a **quantum** version of the classical Bianchi-IX model that looks like a **modified** FRW model.

The main features of this quantum model are:

- the **transformation** of the quantum energy due to anisotropy degrees of freedom into ‘**primordial GW**’,
- a curvature-like term \( kc^2 a^{-2} \) (already present in the classical Bianchi-IX treatment),
- new **repulsive** potential term \( \propto a^{-6} \) generated by our affine CS quantization, responsible for the **resolution** of the singularity.
Conclusions

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Our project consists of the following tasks for the Bianchi IX model:

- Studying classical evolution by dynamical systems method.
- Considering dynamics of model with holonomy corrections implied by LQC.
- Examination of statistics of energy spectrum.
- Rigorous quantization of chaotic dynamics.
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Thank you!