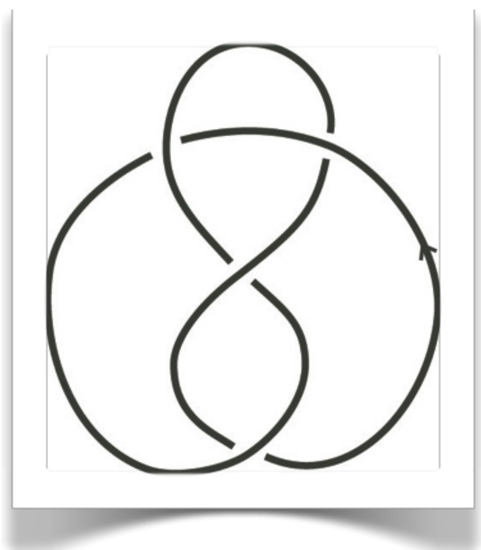
M5-branes, knots, and supersymmetric gauge theories



Piotr Sułkowski

Faculty of Physics, University of Warsaw

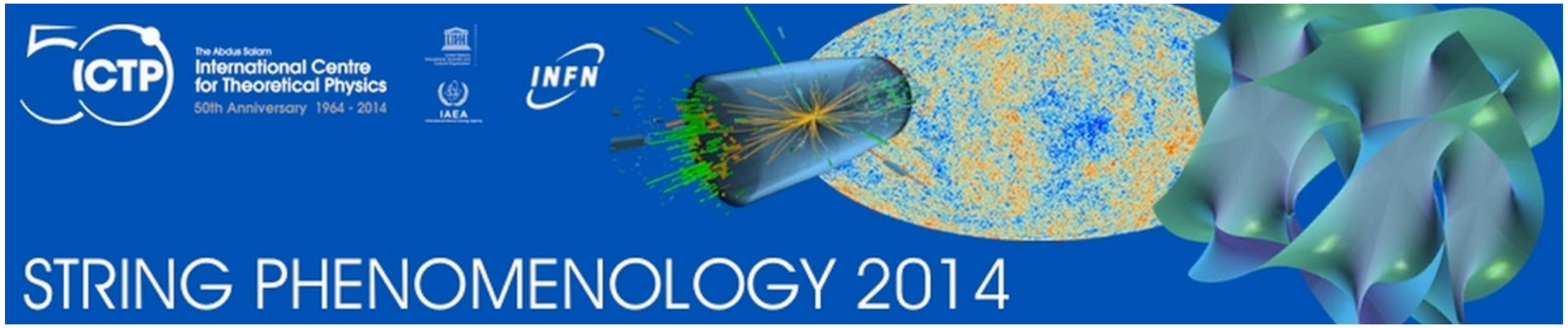
Spała, June 2014

What's new in Strings...

Overview, visionary, and other plenary talks:

Vafa	On 6d CFT's
Tachikawa	Recent advances in SUSY
Stieberger	Unity of tree-level superstring amplitudes
Erdmenger	Applications of AdS/CFT to high energy and condensed matter
Polchinski	Black Hole Information: Spacetime versus Quantum Mechanics
Bizoń	Gravitational turbulent instability of AdS_5
Kovac	Detection of B-mode Polarization at Degree Angular Scales with BICEP2
Moore	Visionary: Physical Mathematics and the Future

What's new in string theory...



What’s new in string theory...

stringtheory.pl/2012

30th Max Born Symposium

stringtheory.pl/2013

Konferencja
5-7 kwietnia 2013
@IF.UJ Kraków

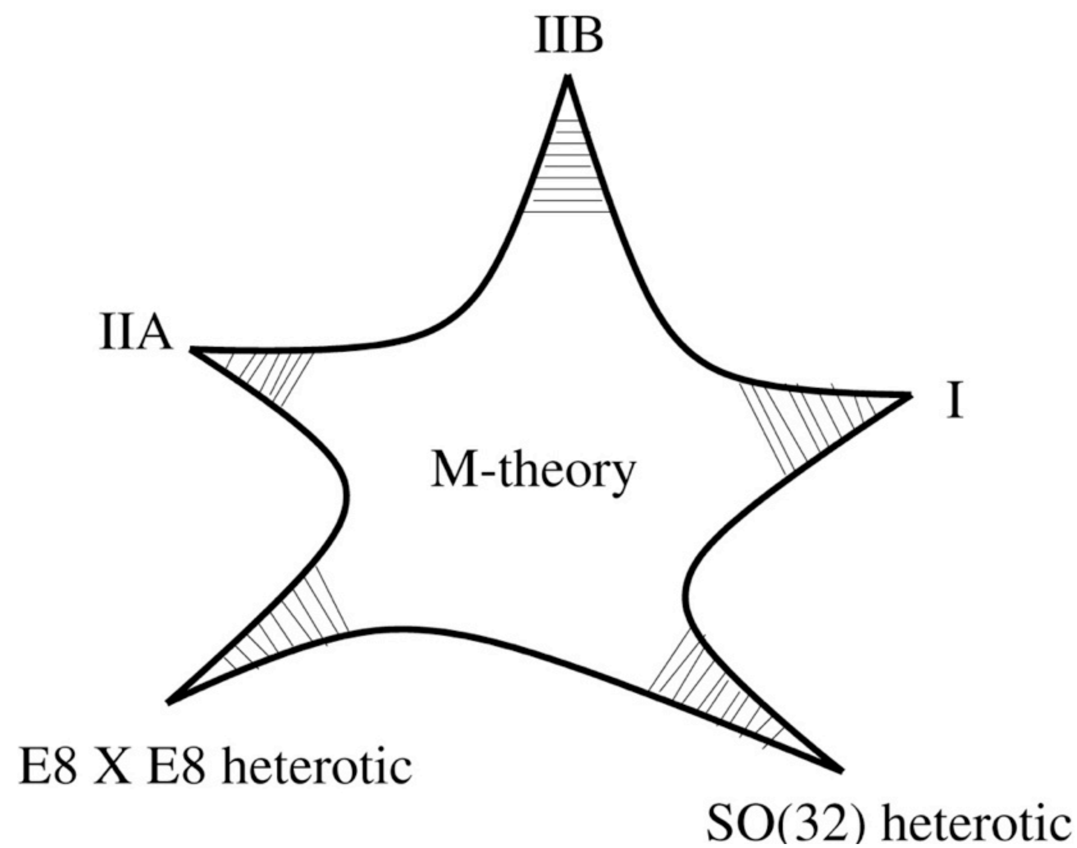
advanced school in string theory

stringtheory.pl/2014

Faculty of Physics | University of Warsaw | Poland

25-27 April

Shortest review of M-theory...



Fundamental objects in M-theory:
- three-form gauge field
- (2+1)-dim **M2-branes** ⟶ **IIA:** fundamental strings, D2-branes
- (5+1)-dim **M5-branes** ⟶ **IIA:** NS5-branes, D4-branes

Objects in M-theory are fundamental and unique!

W. Nahm, Nucl.Phys.B135 (1978) 149

SUPERSYMMETRIES AND THEIR REPRESENTATIONS

W. Nahm

CERN - Geneva

A B S T R A C T

We determine all manifest supersymmetries in more than 1+1 dimensions, including those with conformal or de Sitter space-time symmetry. For the supersymmetries in flat space we determine the structure of all representations and give formulae for an effective computation. In particular we show that at least for masses $m^2 = 0,1,2$ the states of the spinning string form supersymmetry multiplets.

M-theory, M2-branes, and M5-branes

Low energy description of **M-theory:**

11-dim supergravity, identified in Nahm's paper:

Supergravity theories are possible in at most 10+1 dimensions

M-theory, M2-branes, and M5-branes

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Low energy description of multiple **M2-branes:**

ABJM theory (Aharony-Bergman-Jafferis-Maldacena, 2008)

- *three dimensional Chern-Simons-matter theories with gauge groups U(N)xU(N), with explicit N=6 superconformal symmetry*
- *interesting features, e.g. ~* $N^{3/2}$ *degrees of freedom*

M-theory, M2-branes, and M5-branes

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> Supergravity theories are possible in at most 10+1 dimensions

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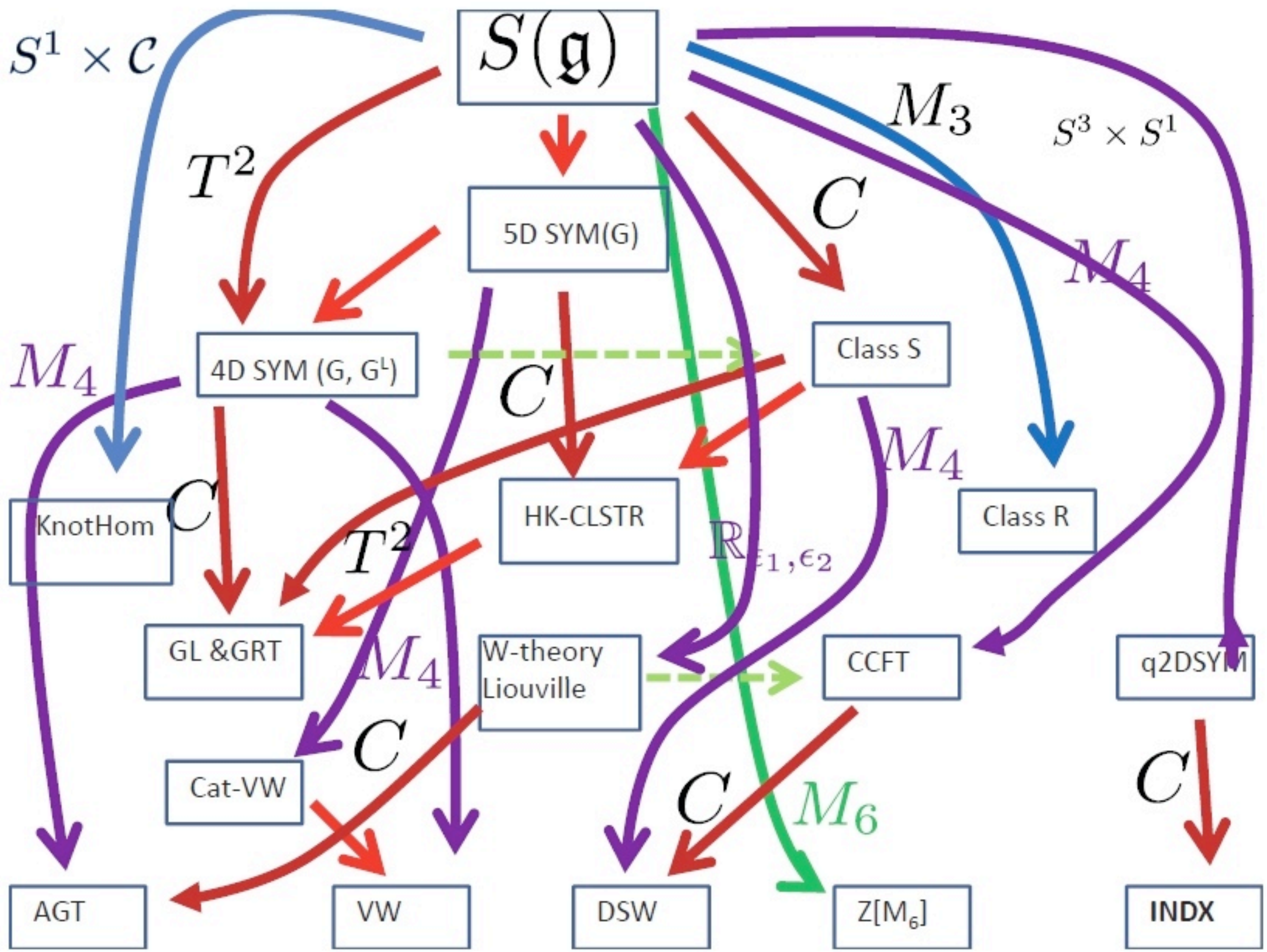
Low energy description of multiple **M5-branes** still unknown, but:

- *involves **6-dim (2,0) superconformal theory**, also identified by Nahm*
- *must imply S-duality in 4-dim, ~ N^3 degrees of freedom, etc.*

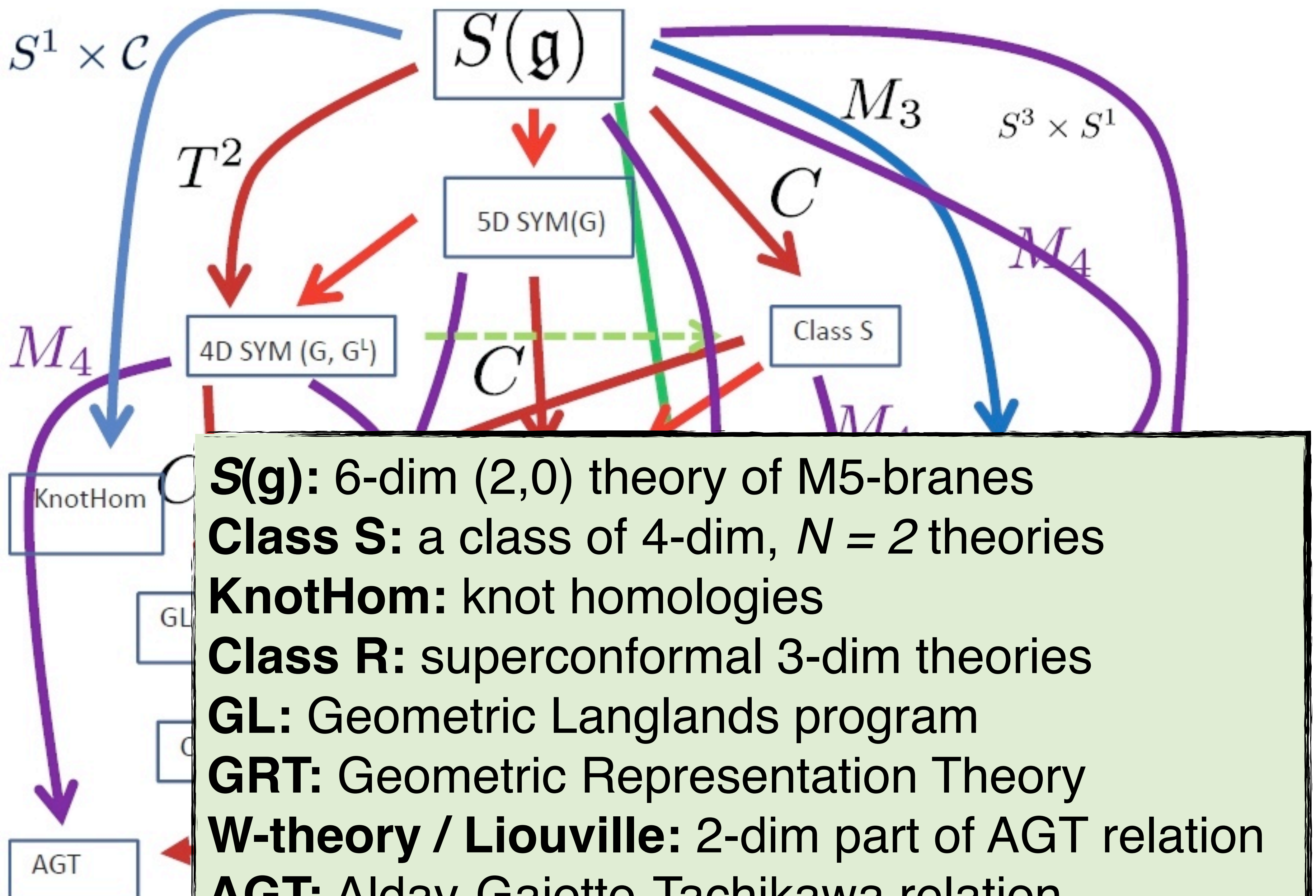
Very important and still very little understood!

M5-branes and a web of dualities



(G. Moore, Felix Klein Lecture, Bonn, 2012)

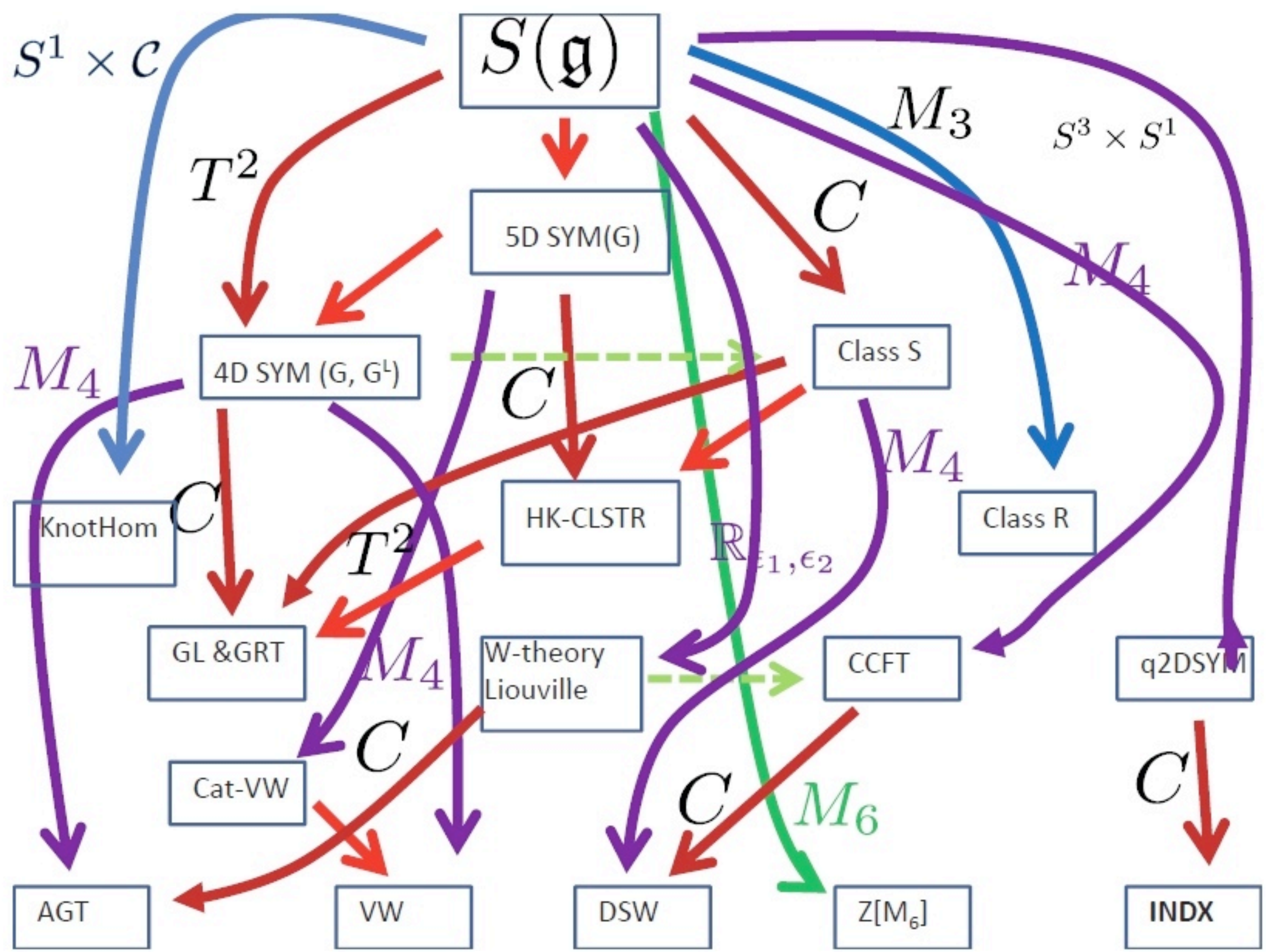
M5-branes and a web of dualities



***S*(g):** 6-dim (2,0) theory of M5-branes
Class S: a class of 4-dim, *N = 2* theories
KnotHom: knot homologies
Class R: superconformal 3-dim theories
GL: Geometric Langlands program
GRT: Geometric Representation Theory
W-theory / Liouville: 2-dim part of AGT relation
AGT: Alday-Gaiotto-Tachikawa relation

(G. Moore, Felix Klein Lecture, Bonn, 2012)

M5-branes and a web of dualities



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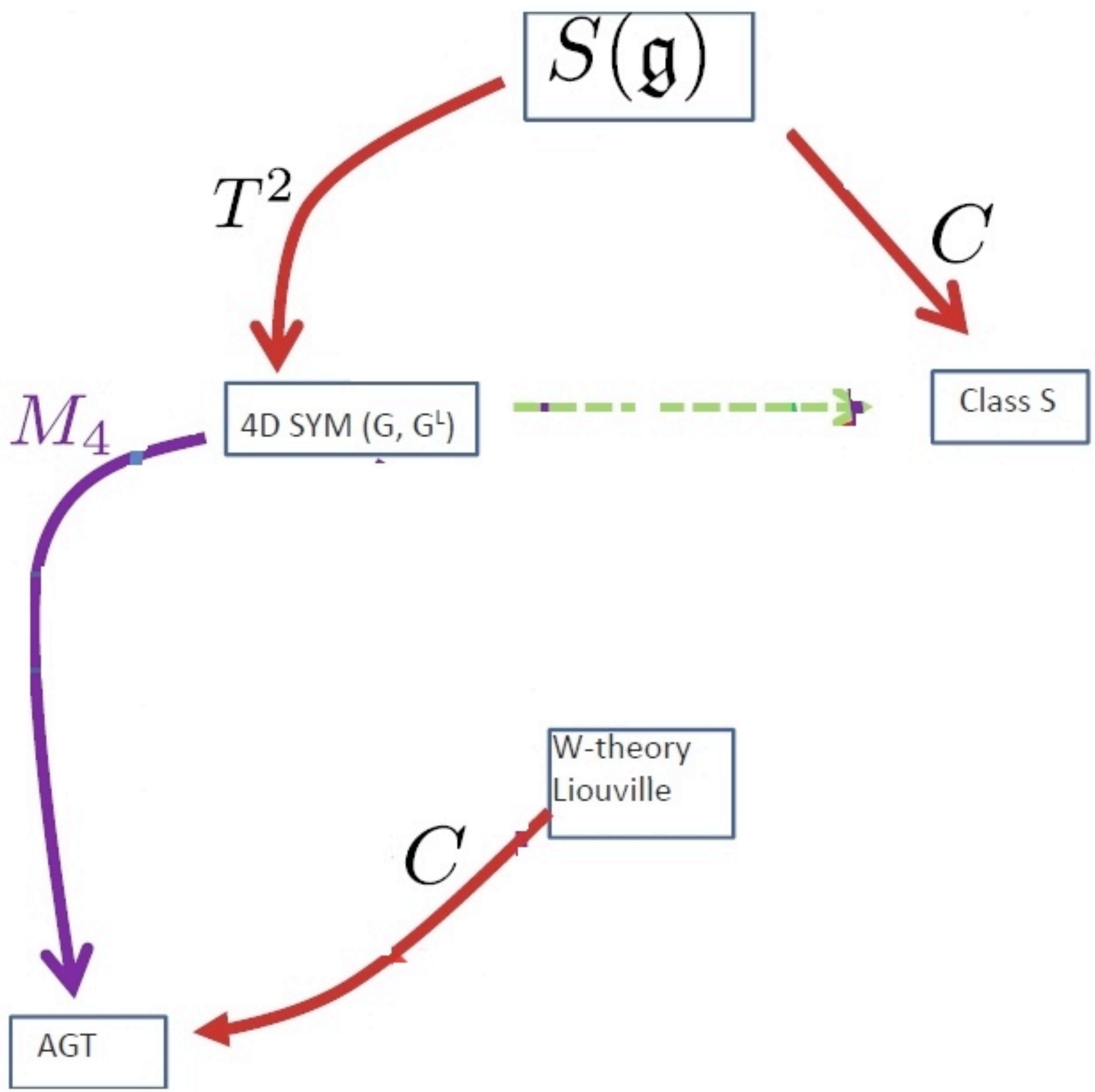
AGT (Alday-Gaiotto-Tachikawa) relation



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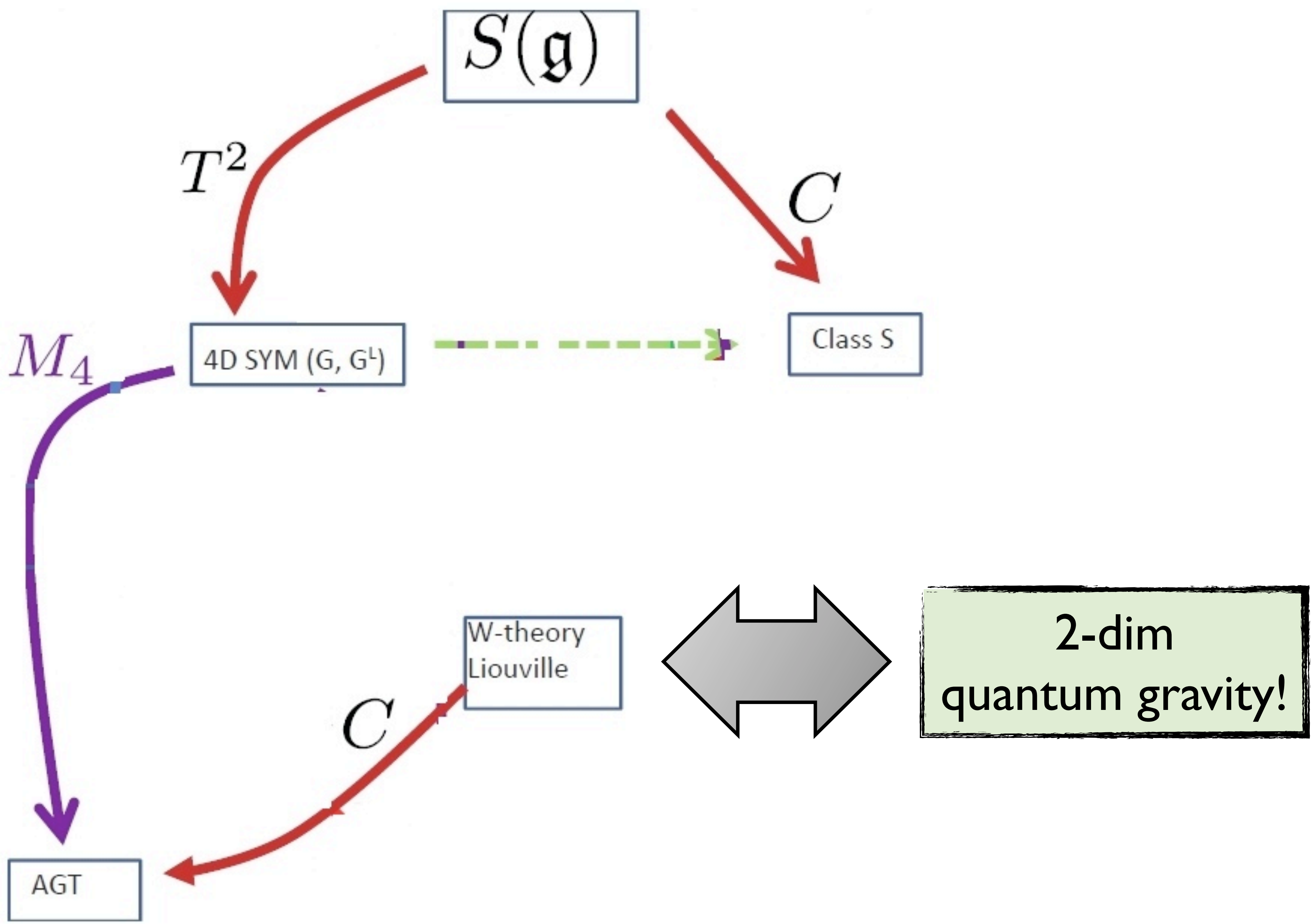

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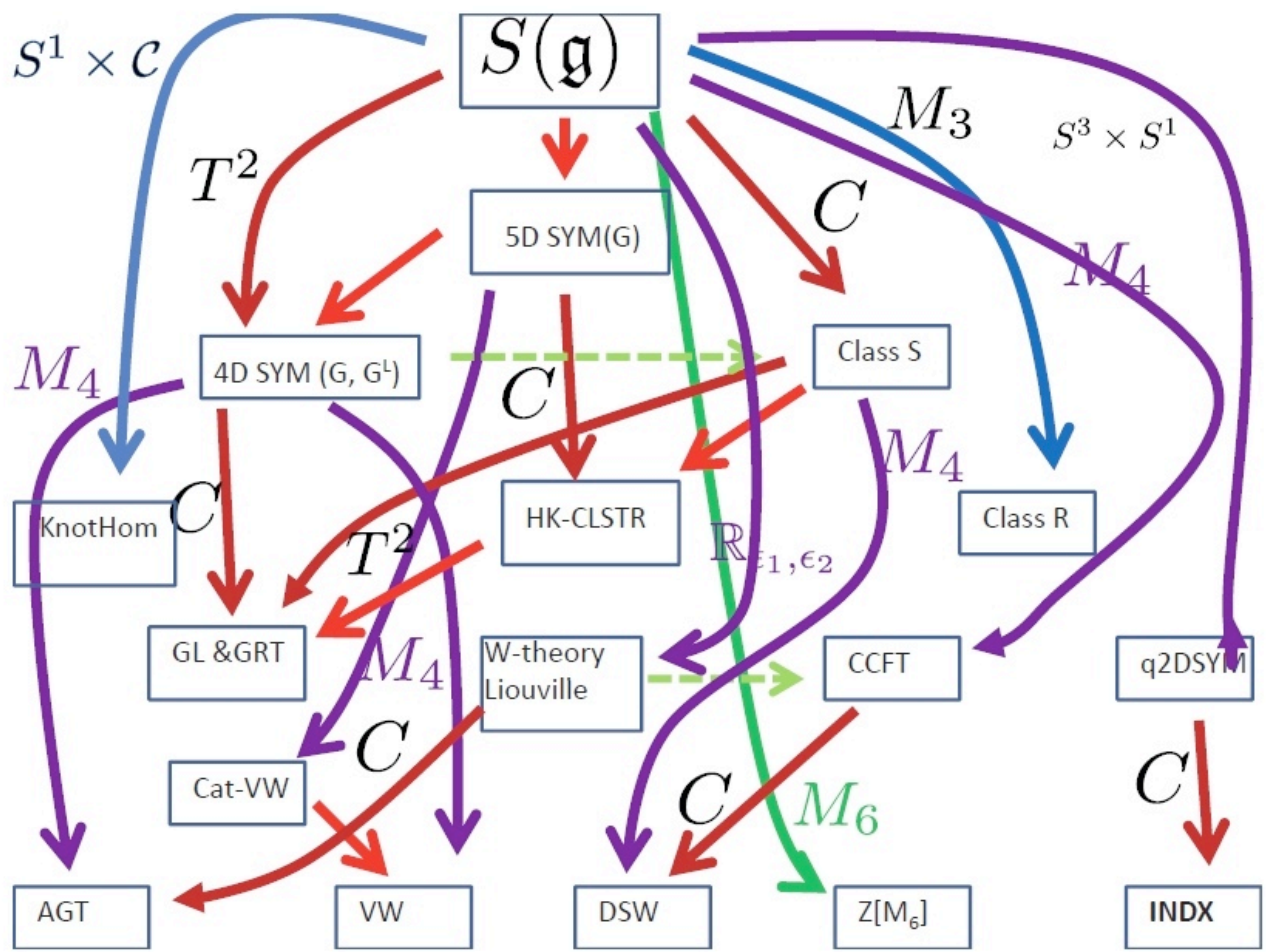
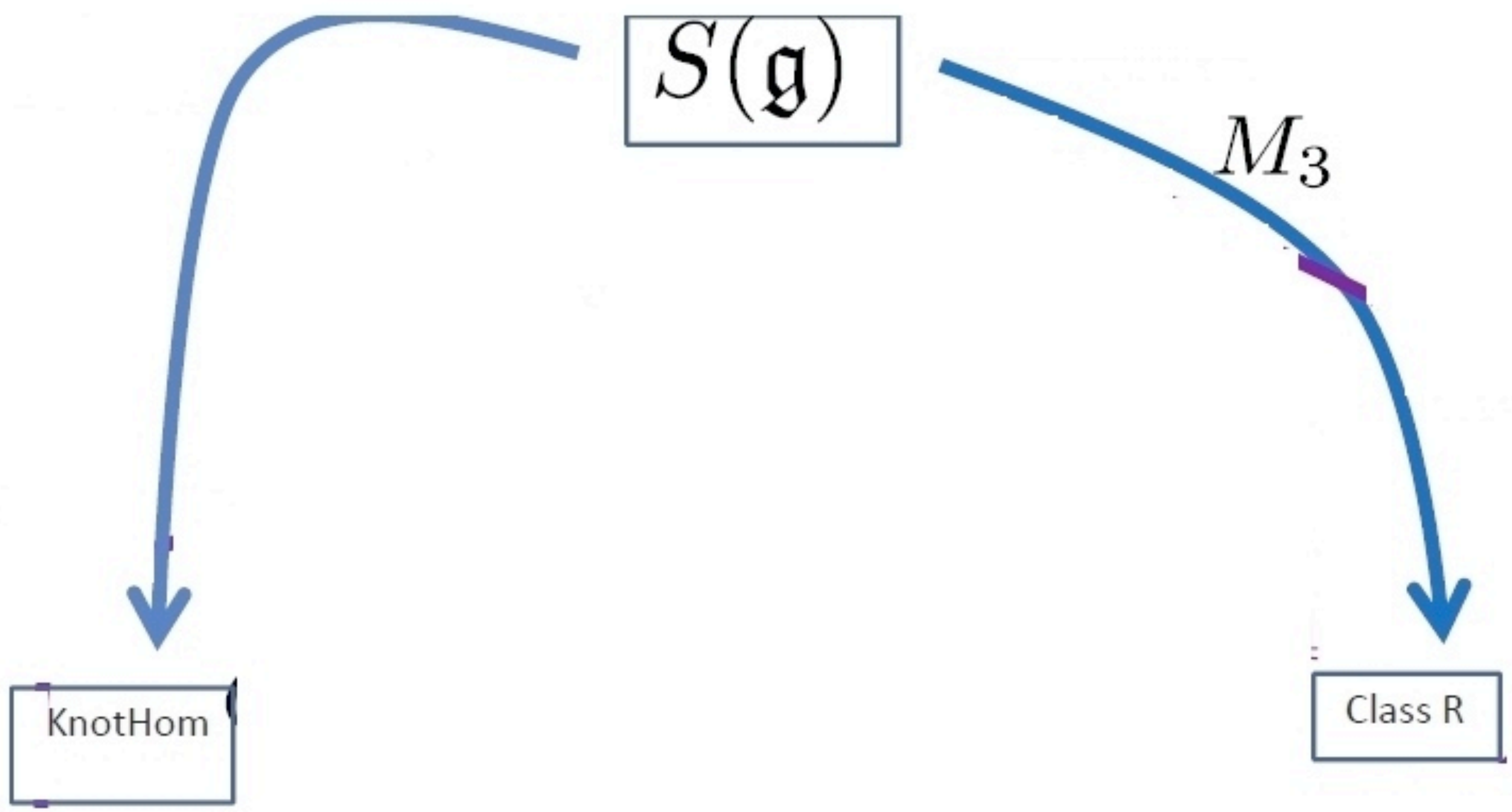


3d-3d correspondence

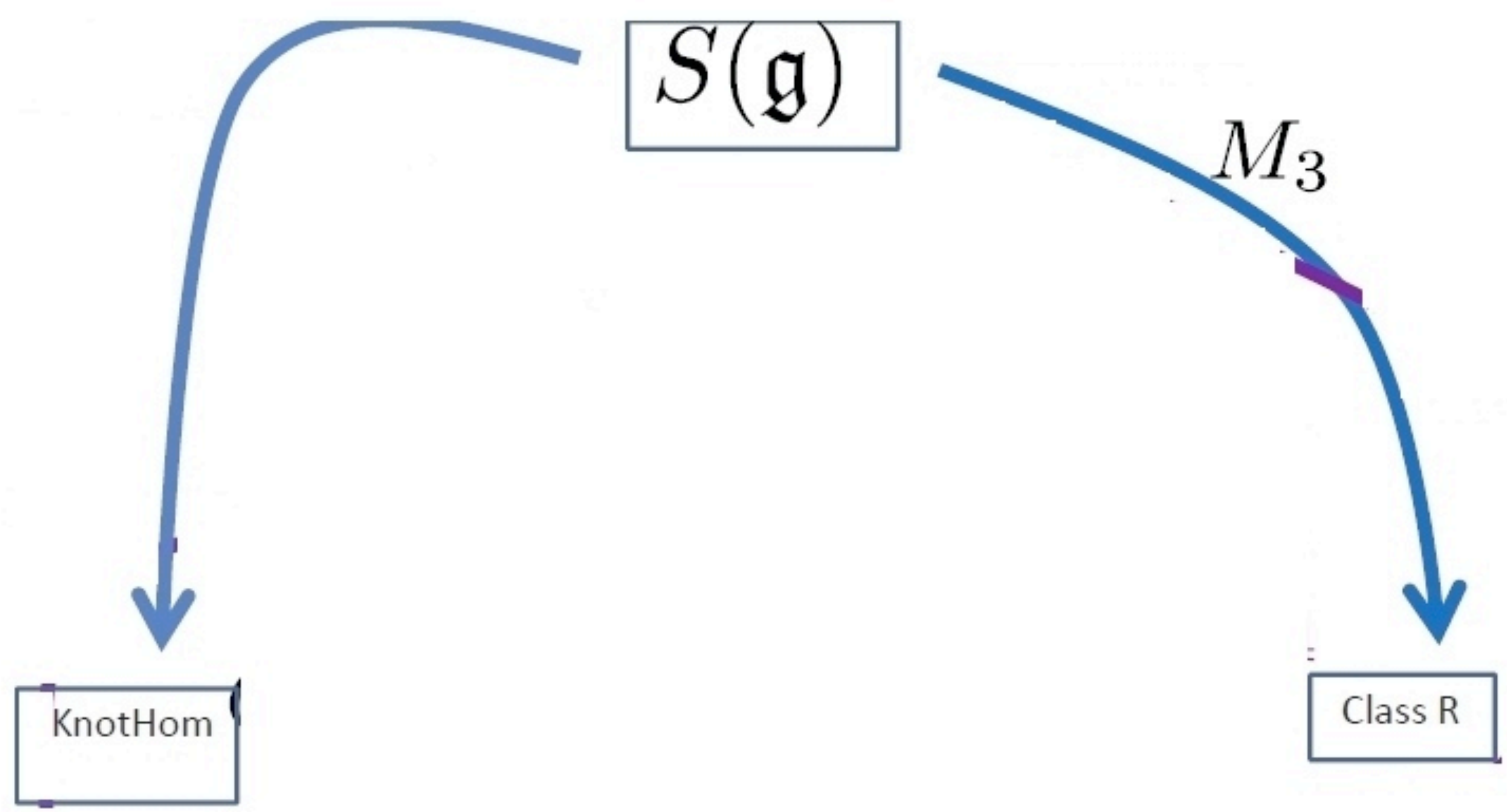


KnotHom:
~ 3-dim Chern-Simons

⟺

Class R:
3-dim *N*=2 theories

3d-3d correspondence



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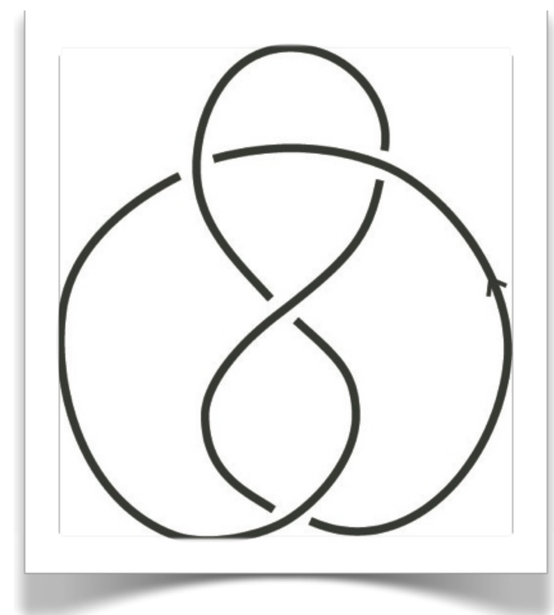
3-dim
quantum gravity!

Chern-Simons theory and knot invariants

$$Z_R(a,q) = \int \mathcal{D}A\Big(\mathrm{Tr}_R e^{\oint A}\Big) e^{\frac{ki}{4\pi}\int \mathrm{Tr}(A\wedge dA + \frac{2}{3}A\wedge A\wedge A)}$$

$$q = e^{\hbar} = e^{\frac{2\pi i}{k+N}}, \quad a = q^N$$



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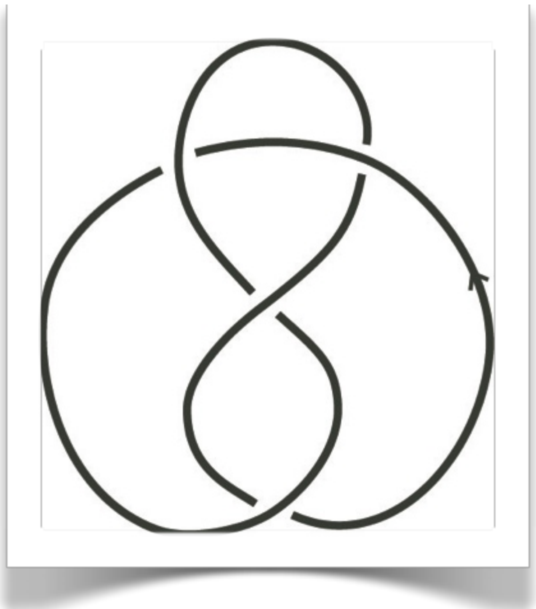
Jones and HOMFLY polynomials



$$J_{\Box}(a,q) = \frac{Z_{\Box}^{SU(N)}(K;a,q)}{Z_{\Box}^{SU(N)}(O;a,q)}$$

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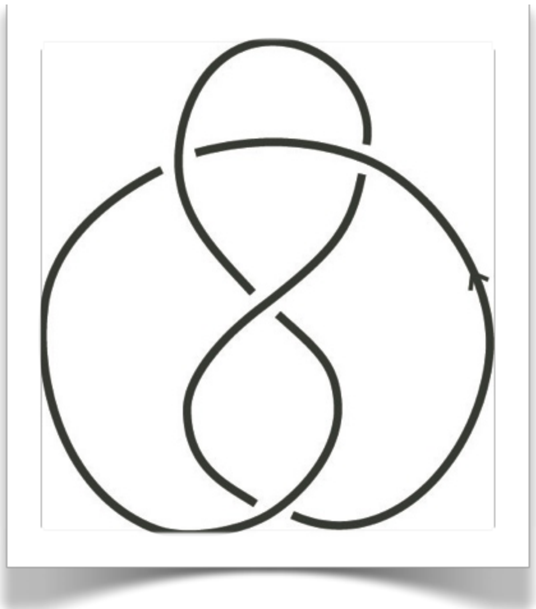

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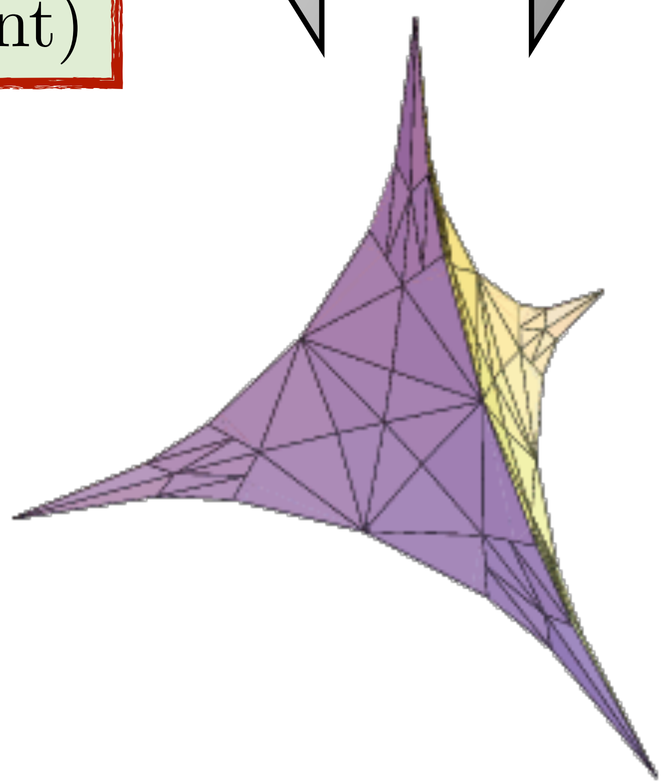
Asymptotics and volume conjecture

$$J_{S^n}(q) \underset{n\to\infty, q^n = x}{\simeq} \exp\left(\frac{1}{\hbar} S_0(x) + S_1(x) + \hbar S_2(x) + \dots \right)$$

3d-3d correspondence

Knot theory:
3-dim Chern-Simons
$S^3 \backslash (\text{Knot complement})$

Class R:
N=2 SUSY theories
on *R*^{3}



$$Z_{Chern-Simons} = J_{S^n} = Z_{SUSY}$$

$$S_0 = \text{twisted superpotential } \widetilde{W}$$

S_0 is a sum of dilogarithms, which are in one-to-one correspondence with
- ideal tetrahedra in a decomposition of the knot complement
- chiral fields in *N*=2 SUSY theory

Generalization: knot homologies

Khovanov homology

$$J_{\square}(q) = \sum_{j,k} q^j (-1)^k \dim \mathcal{H}_{j,k}$$

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Superpolynomial

$$P_R(a,q,t) = \sum_{i,j,k} a^i q^j t^k \dim \mathcal{H}^R_{i,j,k} = \mathrm{Tr}_{\mathcal{H}_{BPS}} a^{\beta} q^P t^F$$

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Claim: 3d-3d relation still holds!

t is a fugacity for some extra U(1) symmetry of *N*=2 theory

$$P_{S^n}(a,q,t) \underset{n\to\infty, q^n = x}{\simeq} \exp\left(\frac{1}{\hbar}\widetilde{W}(x;a,t) + \ldots\right)$$

3d-3d relation and knot homologies

Superpolynomials not (yet?) defined/computed by mathematicians, but predicted by physicists!

$$P_R(a,q,t) = \sum_{i,j,k} a^i q^j t^k \dim \mathcal{H}^R_{i,j,k} = \mathrm{Tr}_{\mathcal{H}_{BPS}} a^\beta q^P t^F$$

Refined Chern-Simons

Differentials in knot homologies

3d-3d relation and brane engineering

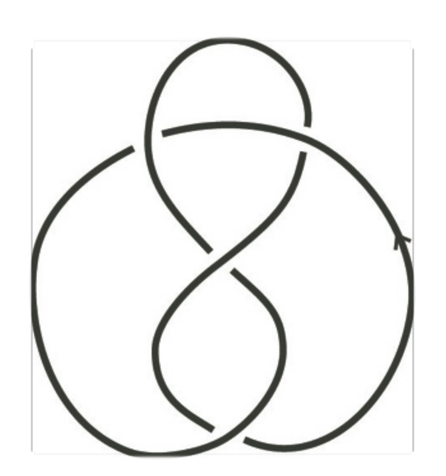
- knot realized as intersection of M5-branes
- effective 3-dim theory on **R^2xR**
- *q* and *t* correspond to two U(1) rotations

$$
\begin{array}{rl}
\text{space-time:} & \mathbb{R}^4 \times T^*\mathbb{S}^3 \times \mathbb{R} \\
 & \cup \\
N \text{ M5-branes:} & \mathbb{R}^2 \times \mathbb{S}^3 \times \mathbb{R} \\
 & || \\
r \text{ M5-branes:} & \mathbb{R}^2 \times L_K \times \mathbb{R}
\end{array}
$$

Example - figure-8 knot, 4_1

$$P_n(\mathbf{4_1}; a, q, t) =$$
$$= \sum_{k=0}^{\infty} (-1)^k a^{-k} t^{-2k} q^{-k(k-3)/2} \frac{(-atq^{-1}, q)_k}{(q,q)_k} (q^{1-n}, q)_k (-at^3 q^{n-1}, q)_k$$

n	$P_n(\mathbf{4_1}; a, q, t)$
1	1
2	$a^{-1}t^{-2} + t^{-1}q^{-1} + 1 + qt + at^2$
3	$a^{-2}q^{-2}t^{-4} + (a^{-1}q^{-3} + a^{-1}q^{-2})t^{-3} + (q^{-3} + a^{-1}q^{-1} + a^{-1})t^{-2} +$ $+(q^{-2} + q^{-1} + a^{-1} + a^{-1}q)t^{-1} + (q^{-1} + 3 + q) + (q^2 + q + a + aq^{-1})t +$ $+(q^3 + aq + a)t^2 + (aq^3 + aq^2)t^3 + a^2q^2t^4$



Example - trefoil knot, 3_1

$$P_n(\mathbf{3_1}; a, q, t) = \sum_{k=0}^{n-1} a^{n-1} t^{2k} q^{n(k-1)+1} \frac{(q^{n-1}, q^{-1})_k (-atq^{-1}, q)_k}{(q, q)_k}$$

$$\begin{aligned} J_\square(q) &= q + q^3 - q^4 \\ J_\square(a,q) &= \frac{a}{q} + aq - a^2 \\ Kh_\square(q,t) &= q + q^3t^2 + q^4t^3 \\ P_\square(a,q,t) &= \frac{a}{q} + aqt^2 + a^2t^3 \end{aligned}$$

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$$P_n(\mathbf{3_1}; a, q, t) \sim \int dz\, e^{\frac{1}{\hbar}\left(\widetilde{\mathcal{W}}(\mathbf{3_1}; z, x) + \mathcal{O}(\hbar)\right)}$$

$$\widetilde{\mathcal{W}}(\mathbf{3_1}; z, x) = -\frac{\pi^2}{6} + (\log z + \log a) \log x + 2(\log t)(\log z)$$
$$+ \mathrm{Li}_2(xz^{-1}) - \mathrm{Li}_2(x) + \mathrm{Li}_2(-at) - \mathrm{Li}_2(-atz) + \mathrm{Li}_2(z)$$

Dual 3-dim, *N*=2 SUSY gauge theory

chiral field ϕ $\longleftrightarrow$ twisted superpotential $\Delta\widetilde{\mathcal{W}} = \mathrm{Li}_2\Big((-t)^{n_F}\prod_i (x_i)^{n_i}\Big)$

Therefore the **spectrum** of the theory can be read off from $\widetilde{\mathcal{W}}$:

$$\widetilde{\mathcal{W}}(\mathbf{3_1}; z, x) = \mathrm{Li}_2(xz^{-1}) - \mathrm{Li}_2(x) + \mathrm{Li}_2(-at) - \mathrm{Li}_2(-atz) + \mathrm{Li}_2(z) + \ldots$$

$\mathbf{3_1}$**knot**	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	parameter
$U(1)_{\text{gauge}}$	-1	0	0	-1	1	z
$U(1)_F$	0	0	1	-1	0	$-t$
$U(1)_Q$	0	0	1	-1	0	a
$U(1)_L$	1	-1	0	0	0	x

Summary

M5-branes

Knot theory, Chern-Simons

3d, $\mathcal{N}$=2 SUSY gauge theory

Foundation for Polish Science

Summary

(Some) literature:

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"3d analogs of Argyres-Douglas theories and knot homologies",
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"Super-A-polynomial for knots and BPS states",
Nucl. Phys. B867 (2013) 506.

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