Response of gravitational wave detectors in metric theories of gravity

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Contents

Signal

plane monochromatic wave with polarization π moving along $\mathbf{\Omega}$

$$e^{i\omega(t-\mathbf{\Omega}\cdot\mathbf{x})}\boldsymbol{\epsilon}^{\pi}$$

$$\pi \;=\; sl, st \text{ (scalar)}, \quad vx, vy \text{ (vector)}, \quad tp, tc \text{ (tensor)}$$

$\boldsymbol{\epsilon}^{\pi}$ are the polarization tensors with components:

$$\begin{aligned}
\tilde{\epsilon}^{sl} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \tilde{\epsilon}^{st} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\tilde{\epsilon}^{vx} &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \tilde{\epsilon}^{vy} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
\tilde{\epsilon}^{tp} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tilde{\epsilon}^{tc} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\end{aligned}$$

Detector response

$\Delta T_{aba}(t)$ – time of flight: a⟶b⟶a

$$\frac{\Delta T_{aba}(t)}{T} = \underbrace{\mathcal{T}(\omega L, \mathbf{\Omega} \cdot \boldsymbol{n}_{ab})\, F^{\pi}(\boldsymbol{n}_{ab})}_{H(\omega L, \mathbf{\Omega}) \text{ – detector response}} \; e^{i\omega(t - \mathbf{\Omega} \cdot \boldsymbol{x}_a)}$$

$$\mathcal{T}(x, c) = \frac{1}{2}\left[\operatorname{sinc}\left[\frac{x}{2}(1+c)\right] e^{-\frac{i x}{2}(1+c)} + \operatorname{sinc}\left[\frac{x}{2}(1-c)\right] e^{-\frac{i x}{2}(3+c)}\right] \qquad \text{frequency transfer}$$

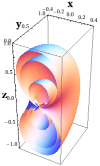
$$F^{\pi}(\boldsymbol{n}) = \frac{1}{2}\mathbf{n} \otimes \mathbf{n} : \epsilon^{\pi} \qquad \text{angular pattern}$$

Detector response = frequency transfer × angular pattern

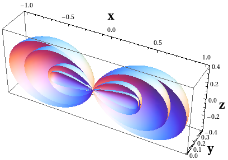
Frequency transfer

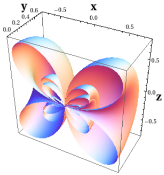
Frequency transfer × angular pattern   (detector arm along **x**)

st

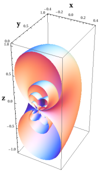


sl





v



t

Monochromatic gravitational wave

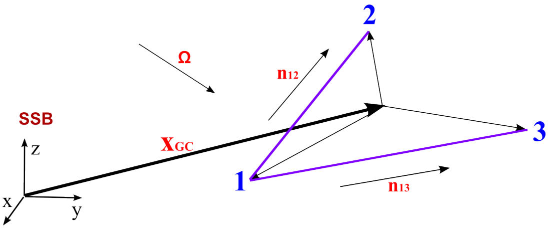
- tensor, vector

$$\mathbf{h}(\mathbf{x},t) = h_0^{\pi 1}\cos[\omega(t-\boldsymbol{\Omega}\cdot\mathbf{x})+\phi_0]\boldsymbol{\epsilon}^{\pi 1} + h_0^{\pi 2}\sin[\omega(t-\boldsymbol{\Omega}\cdot\mathbf{x})+\phi_0]\boldsymbol{\epsilon}^{\pi 2}$$

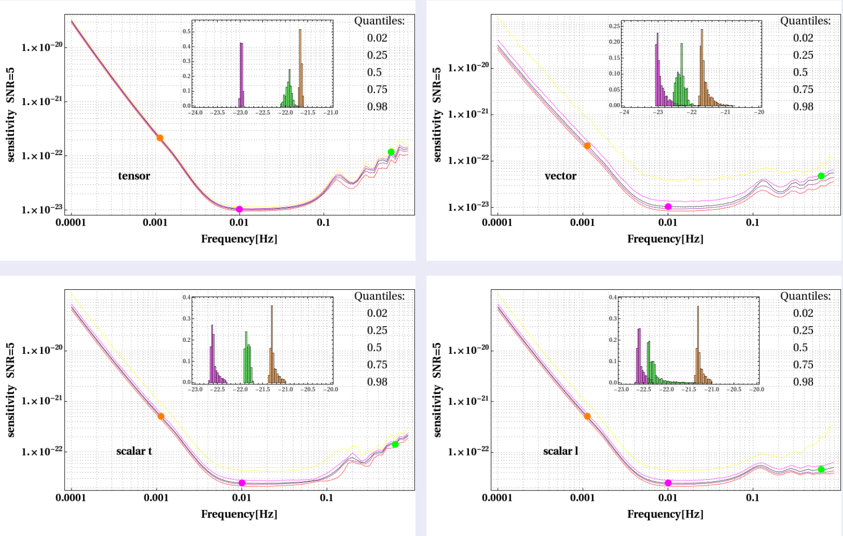
- scalar t, scalar l

$$\mathbf{h}(\mathbf{x},t) = h_0^{\pi}\cos[\omega(t-\boldsymbol{\Omega}\cdot\mathbf{x})+\phi_0]\boldsymbol{\epsilon}^{\pi}$$

Triangular configuration:

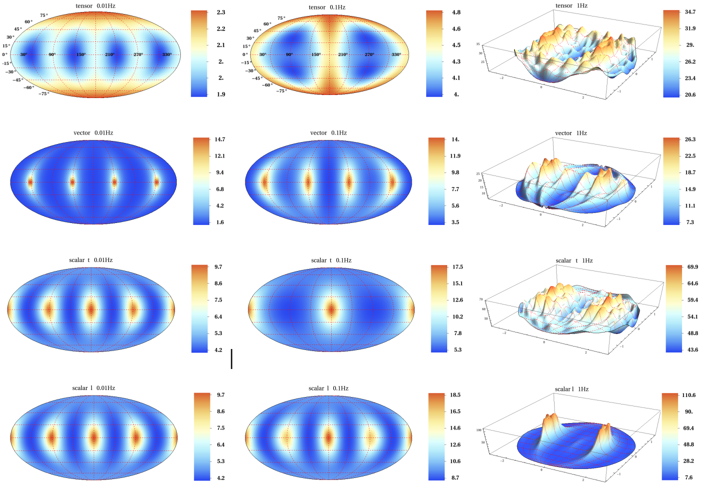


Monochromatic gravitational wave



eLISA, X; uniform distribution of sky location and $\langle$orientations$\rangle$.

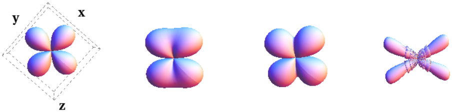
Sensitivity: sky map; SNR $= 5$; 10mHz



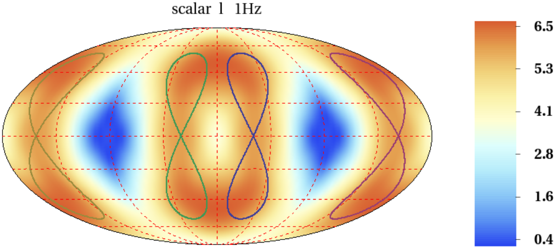
eLISA, X; sensitivity $[10^{-23}]$; $\langle$uniform distribution of orientations$\rangle$.

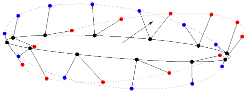
SNR for sl mode

sl response for eLISA arms:



SNR ($h = 10^{-22}$); uniform distribution of orientations.





Triangular configuration - stochastic signal

- stochastic background: self- and cross- correlations

$$\langle s_I(t) s_J(t) \rangle = \frac{1}{2} \int_{-\infty}^{\infty} \int_{S^2} S_h(\omega, \mathbf{\Omega}) \mathcal{F}_{IJ}(\omega, \mathbf{\Omega}) \times \text{filter} \times \frac{d\Omega}{4\pi} d\omega,$$

$S_h(\omega, \mathbf{\Omega})$ - possibly anisotropic background

- antenna pattern

$$\mathcal{F}_{IJ}(\omega, \mathbf{\Omega}) = e^{i\omega\, \mathbf{\Omega} \cdot (\mathbf{x}_J - \mathbf{x}_I)} \left[\mathcal{T}_I(\omega L, \mathbf{\Omega} \cdot \boldsymbol{n}_I) F^{\pi}(\boldsymbol{n}_I) \right] \left[\mathcal{T}_J(\omega L, \mathbf{\Omega} \cdot \boldsymbol{n}_J) F^{\pi}(\boldsymbol{n}_J) \right]^*$$

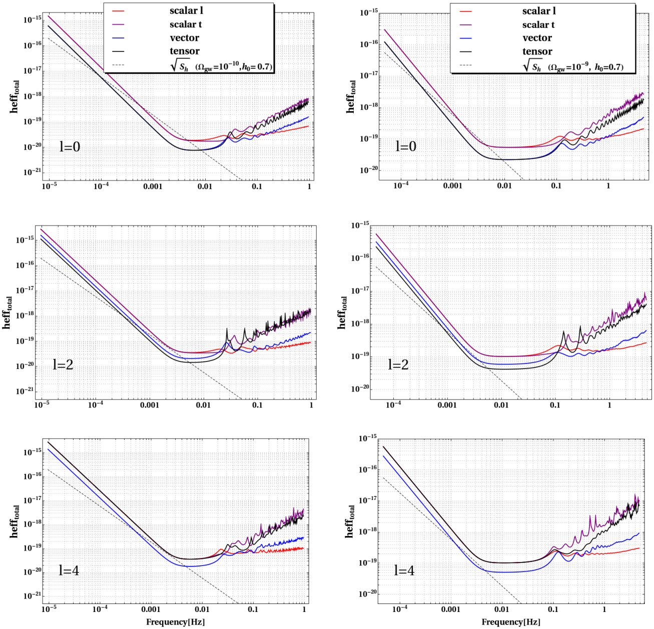
- spherical harmonic decomposition of $\mathcal{F}_{IJ}$: $a_{lm}(\omega L) = \int_{S^2} Y^*_{lm}(\mathbf{\Omega}) \mathcal{F}(\omega L, \mathbf{\Omega}) \frac{d\,\Omega}{4\pi}$

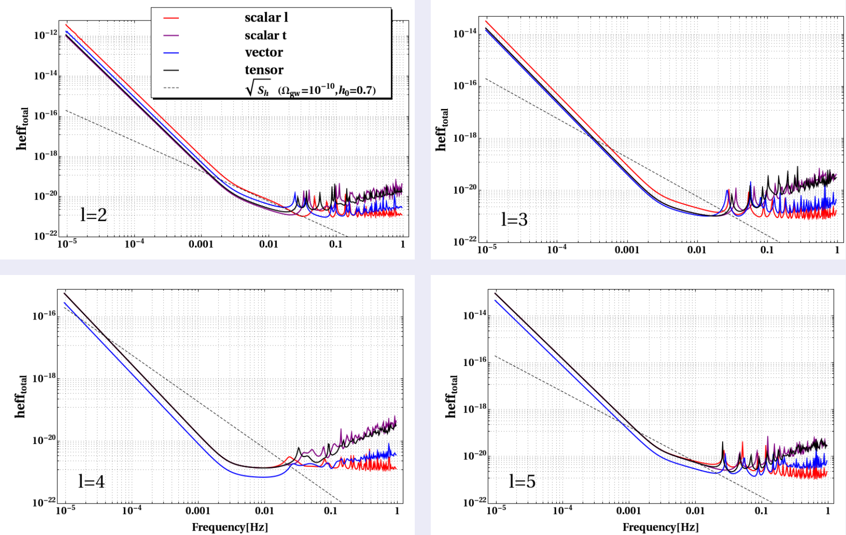
angular power of the antenna pattern: $$\sigma_l(\omega L) = \sqrt{\frac{1}{2l+1} \sum_{m=-l}^{l} |a_{lm}(\omega L)|^2}$$

- sensitivity

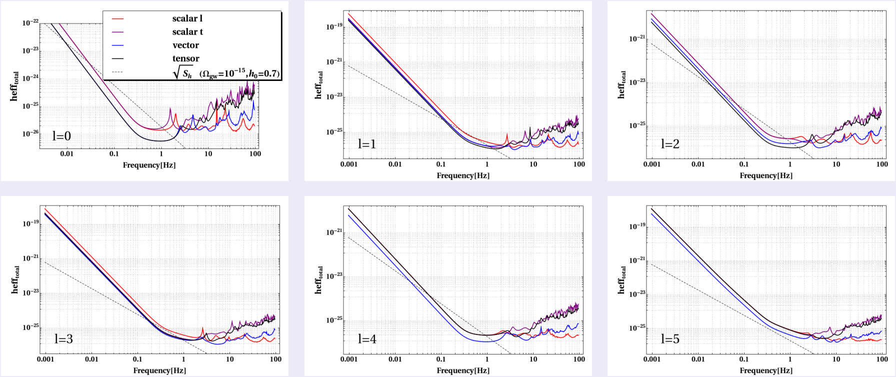
self-correlations: $h_{eff} = (4\pi)^{1/4} \sqrt{\frac{S_n(f)}{\sigma_I(f)}}$, cross-correlations: $h_{eff} = \left(\frac{4\pi}{2\,T\,f} \right)^{1/4} \left[\frac{S_{n_I}(f) S_{n_J}(f)}{\sigma_I^2(f)} \right]^{1/4}$

self-corr: LISA: AA eLISA: XX





cross-corr: **AE**, $T = 1$yr



cross-corr: **AA** + **AE** + $\cdots$ + **TT**, $T = 1$yr

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