Localized configurations in the AdS/CFT correspondence

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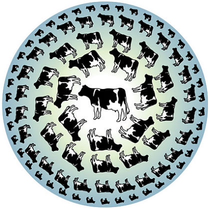
- **Holographic principle**
  Quantum gravity in $d+1$ dimensions must have a number of d.o.f. which scales like that of QFT in d dimensions

  't Hooft and Susskind '93



- String Theory realization *AdS/CFT correspondence*
  Solutions of Einstein equations in $d+1$ dimensions $\longleftrightarrow$ States in strongly coupled QFT in d dimensions

  Maldacena '97

- Fascinating links between GR and QFT!
- Physically motivated interesting questions can be addressed by GR methods

The AdS space

- The Anti-de-Sitter spacetime can be given in a Poincare path coordinates

$$ds^2 = \frac{\eta_{\mu\nu} dx^\mu dx^\nu + dz^2}{z^2} \tag{1}$$

where $z > 0$ and boundary is at $z = 0$

- Metric (1) solves $(d+1)$-dim. Einstein equations

$$R_{ab} - \frac{1}{2} g_{ab} R - \frac{d(d-1)}{2} g_{ab} = 0 \tag{2}$$

- This corresponds to the vacuum state in field theory

$$\langle T_{\mu\nu} \rangle = 0$$

- In this talk $d = 4$ i.e. $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence $\longrightarrow$ motivated by applications to solid state physics

Field - Operator correspondence

- Field in the bulk $\longleftrightarrow$ Local operator on the boundary

$$\phi(x,z) \longleftrightarrow \mathcal{O}(x)$$

- Solve Einstein+matter equations with an arbitrary Dirichlet boundary condition $\phi(x, z=0) = \phi_0(x)$
- $\phi_0(x)$ sources operator $\mathcal{O}(x)$
- The near boundary $(z \to 0)$ expansion

$$\phi(x,z) \sim \phi_0(x) z^{d-\Delta} + \phi_1(x) z^{\Delta} + \dots \tag{3}$$

where $\mathrm{dim}[\mathcal{O}] = \Delta$

- The expectation value is the subleading term

$$\phi_1(x) = \langle \mathcal{O}(x) \rangle \tag{4}$$

Example: AdS$_4$ black hole

- Equilibrium state in field theory $\longleftrightarrow$ black hole in the bulk
  Temperature in field theory $\longleftrightarrow$ Hawking temperature
- Solve for metric $\rightarrow$ read off energy density

$$g_{tt}(z) \sim -1 + z^3 g_{tt}^{(3)} + \ldots \tag{5}$$

with

$$\varepsilon_{\mathrm{CFT}} = \langle T_{tt} \rangle = \frac{3N^2}{8\pi} g_{tt}^{(3)} = \frac{16 N^2 \pi^3 T^3}{27} \tag{6}$$

which is determined by conformal symmetry

- To compare free massless bosons

$$\varepsilon_{\mathrm{free}} = \frac{T^3 N^2 \zeta(3)}{\pi} \tag{7}$$

I. R. Klebanov, A. A. Tseytlin, Nucl. Phys. B **475**, 164 (1996)

Scalar field in AdS$_4$/CFT$_3$

- On top of that one can add matter fields to model physical systems
- Scalar field of mass $m^2 = -2$ has the near-boundary expansion

$$\phi(x,z) \sim \phi_0(x) z + \phi_1(x) z^2 + \ldots \tag{8}$$

- Deformation of the theory by an operator $\mathcal{O}(x)$ of conformal dimension $\Delta = 2$, with the source term $\phi_0(x)$

$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}_3} + \int d^3x \phi_0(x) \mathcal{O}(x) \tag{9}$$

and the expectation value $\phi_1(x) = \langle \mathcal{O}(x) \rangle$

Dirac δ-like source

- Consider local line-like source

$$\phi_0(x) = \eta\delta(x) \tag{10}$$

- Source $\phi_0(x) = \cos(kx)$ was used to mimic a lattice
  G. T. Horowitz, J. E. Santos and D. Tong, JHEP **1207**, 168 (2012)
- Try to obtain a lattice $\sum_n \delta(x - na) \rightarrow$ holographic version of the Kronig-Penney model
- Physics of single defects $\rightarrow$ both line-like and point-like

Goal:
Develop techniques to numerically solve Einstein's equations with boundary conditions (10)

Discontinuous boundary conditions in AdS/CFT

- $\theta(x)$ and $m = 0 \rightarrow$ Janus solutions (analytical, 1D ODE)
  D. Bak, M. Gutperle and S. Hirano, JHEP **0305**, 072 (2003)

- $\theta(x)$ and $m = 0$ at $T > 0 \rightarrow$ Janus black holes in $d = 2 + 1$ (numerical PDE, analytical)
  D. Bak, M. Gutperle and R. A. Janik, JHEP **1110**, 056 (2011)

- $\delta(x)$ and $m^2 = -2$ with SUSY $\rightarrow$ analytical and scale invariant
  E. D'Hoker, J. Estes, M. Gutperle and D. Krym, JHEP **0906**, 018 (2009)

Linearized solution

- Line defect linearized solution around empty AdS_4

$$\phi_0(x,z) = \frac{\eta z^2}{\pi(x^2+z^2)} \tag{11}$$

- Easy numerical generalization to finite temperature/chemical potential
- For the $m^2 = -2$ case linearized solution preserves the $1+1$ conformal symmetry along the defect, where the operator is sourced
- Define new variable $\tan\alpha = x/z$ then

$$\phi_0(x,z) = \frac{\eta}{\pi}\cos^2(\alpha) \tag{12}$$

and use it for the backreacted solution at $T = 0$

Single defect backreacted solution at $T = 0$

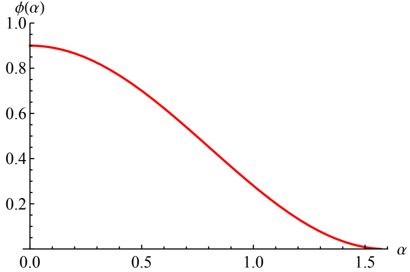
- Assume full solution depends only on α
- The metric will have an AdS slicing

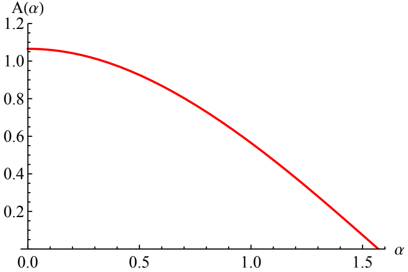
$$ds^2 = \frac{1}{A(\alpha)^2}\left(d\alpha^2 + \frac{dr^2 - dt^2 + dy^2}{r^2}\right) \qquad \phi = \phi(\alpha) \quad (13)$$

- AdS boundaries at both sides of the defect will appear at $\alpha = \pm\alpha_0$ with $\alpha_0 \neq \pi/2$
- We found both numerical and perturbative analytical solutions parametrized by $\phi(\alpha = 0)$

Single defect at $T = 0$

- $\partial_\alpha \phi(0) \neq 0 \rightarrow$ dynamically generated sources for the scalar!
- Dynamically generated scale $\rightarrow$ conformal invariance breaking
- No reduction to an ODE system $\rightarrow$ logarithmic terms !?





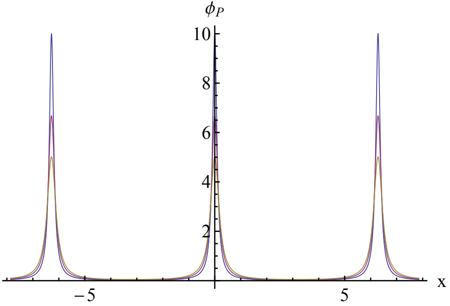
Metric and scalar field for $\phi(0) = 0.9$ with $N = 47$ spectral points

Regularized lattice

- Consider approximation of the Dirac δ lattice at $T > 0$ using linearized vacuum solution

$$\phi_P(x,z) = \sum_{n\in\mathbb{Z}} \phi_0(x+n,z) = \frac{\eta z \sinh(2\pi z)}{\cosh(2\pi z) - \cos(2\pi x)} \qquad (14)$$

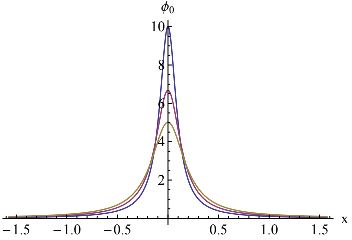
- Model of atomic lattice
- Very large gradients $\rightarrow$ numerically very difficult!



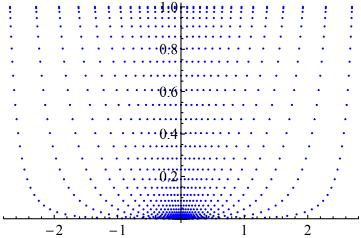
$z_0 = 0.1,\ 0.15,\ 0.2$

Grid modifications

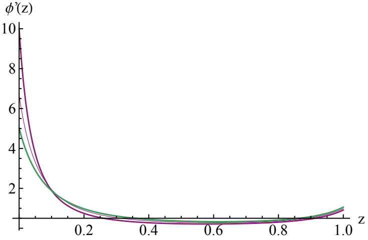
- Analyze the structure of the linearized solution for the Dirac δ function

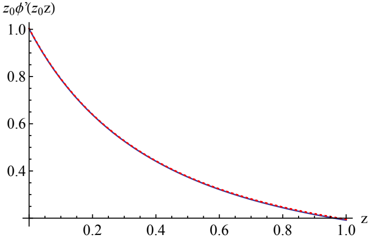


- Use this to adopt the grid points for numeric
- Analyze regularized solutions around the limiting case



- We observe that solutions depend on a scaling variable $w = \frac{z}{z_0}$
- In $z_0 \to 0$ limit only scale invariant derivatives appear $w\partial_w$
- Asymptotic $w \to \infty$ should carry information about the proper boundary conditions

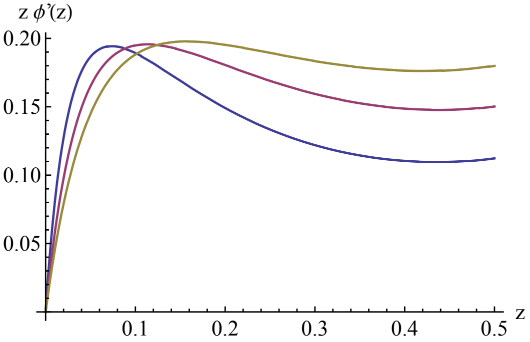




Logarithmic behavior

We can observe evidence for logarithmic behavior of the backreacted Dirac-δ lattice solution

$$\lim_{z\to 0} z\partial_z\phi \neq 0 \tag{15}$$



Summary

- Example of a novel setup in Numerical Relativity
- Development of new numerical techniques to handle the problem $\rightarrow$ e.g. modified grids in spectral methods
- Detailed understanding of boundary conditions
- Holographic model of a lattice $\rightarrow$ solid state physics e.g. optical conductivity, heat transport
- Work in progress ...