Semiclassical limit and transfer matrix in Causal Dynamical Triangulation

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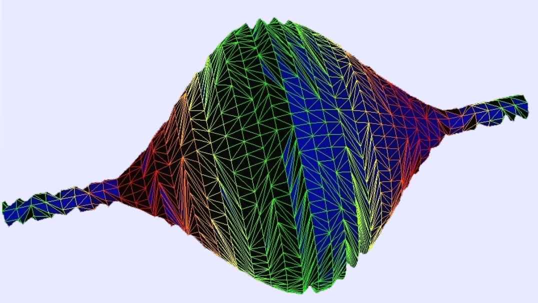
Outline

What is Causal Dynamical Triangulation?

Causal Dynamical Triangulation (CDT) is a background independent approach to quantum gravity.

$$\int \mathrm{D}[g] e^{i S^{EH}[g]} \quad \rightarrow \quad \sum_{\mathcal{T}} e^{-S^{R}[\mathcal{T}]}$$

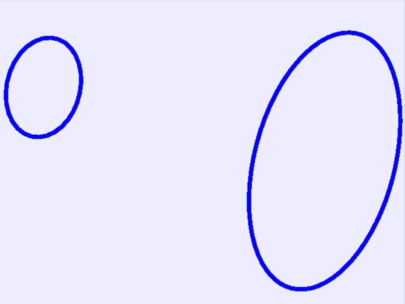
CDT provides a lattice regularization of the formal gravitational path integral via a sum over causal triangulations.



Path integral formulation of quantum gravity

- **Quantum mechanics** can be formulated in terms of *path integrals*: All possible trajectories contribute to the transition amplitude.
- **General Relativity**: gravity is encoded in space-time geometry.
- The role of a trajectory plays now the geometry of four-dimensional space-time.
- All space-time histories contribute to the transition amplitude.

1+1D Example: State of system: one-dimensional spatial geometry



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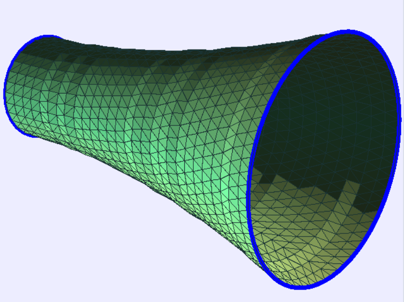
1+1D Example: Evolution of one-dimensional closed universe



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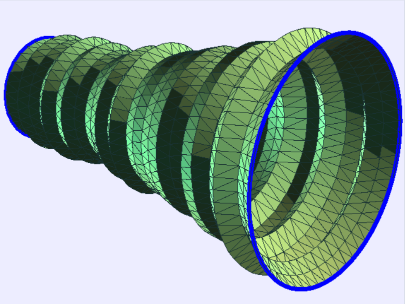
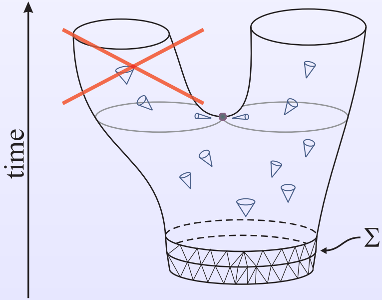
Sum over all two-dimensional surfaces joining the in- and out-state



Causality

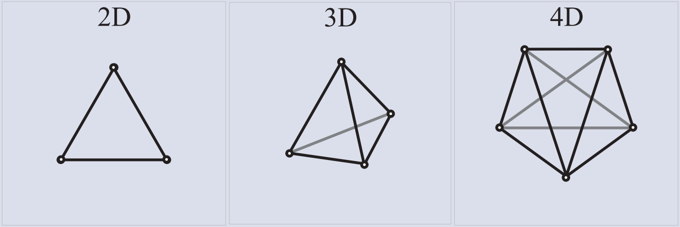
- **Causal D**ynamical **T**riangulations assume global proper-time foliation. Spatial slices (leaves) have fixed topology and are not allowed to split in time.
- Foliation distinguishes between time-like and spatial-like links.
- Such setup does not introduce causal singularities, which lead to creation of baby universes. (EDT)
- CDT defines the class of admissible space-time geometries which contribute to the transition amplitude.



Regularization by triangulation

- **Discretization** is a the standard regularization used in QFT.
- 4D simplicial manifold is obtained by gluing pairs of 4-simplices along their 3-faces.
- Spatial states are 3D geometries with a topology S^3. Discretized states are build from equilateral **tetrahedra**.
- The metric is **flat** inside each 4-simplex.
- Length of time links a_t and space links a_s is constant.
- Curvature is localized at triangles.

Fundamental building blocks of *Euclidean DT*



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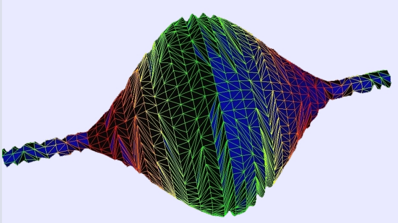
4D space-time with topology $S^3 \times S^1$



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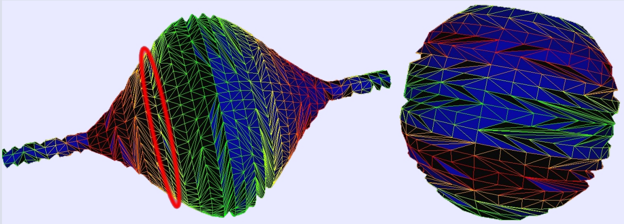
3D spatial slices with topology S^3



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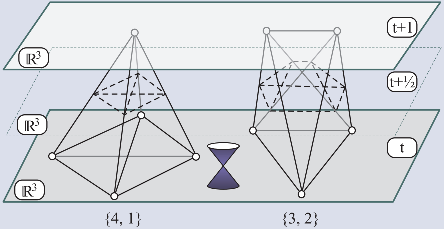
Fundamental building blocks of 4D CDT



Regge action

The Einstein-Hilbert action has a natural realization on piecewise linear geometries called Regge action

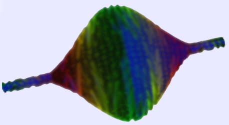
$$S^E[g] = -\frac{1}{G}\int \mathrm{d}t \int \mathrm{d}^D x \sqrt{g}(R - 2\Lambda)$$

N_0 number of vertices

N_4 number of simplices

N_{14} number of simplices of type $\{1,4\}$

K_0 K_4 Δ bare coupling constants $(G, \Lambda, a_t/a_s)$



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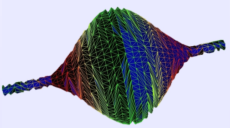

Causal Dynamical Triangulations

- The partition function of quantum gravity is defined as a formal integral over all geometries weighted by the Einstein-Hilbert action.

$$Z = \int \mathrm{D}[g] e^{iS^{EH}[g]}$$

- To make sense of the gravitational path integral one uses the standard method of regularization - discretization.
- The path integral is written as a nonperturbative sum over all causal triangulations $\mathcal{T}$.
- Wick rotation is well defined due to global proper-time foliation. $(a_t \to i a_t)$
- Using **Monte Carlo** techniques we can approximate expectation values of observables.

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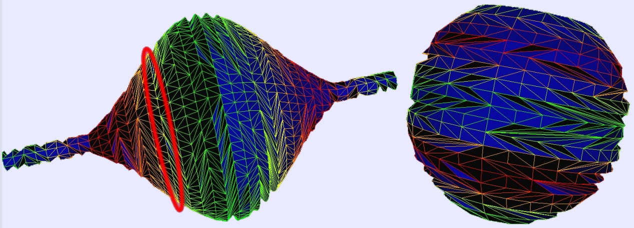
Spatial slices

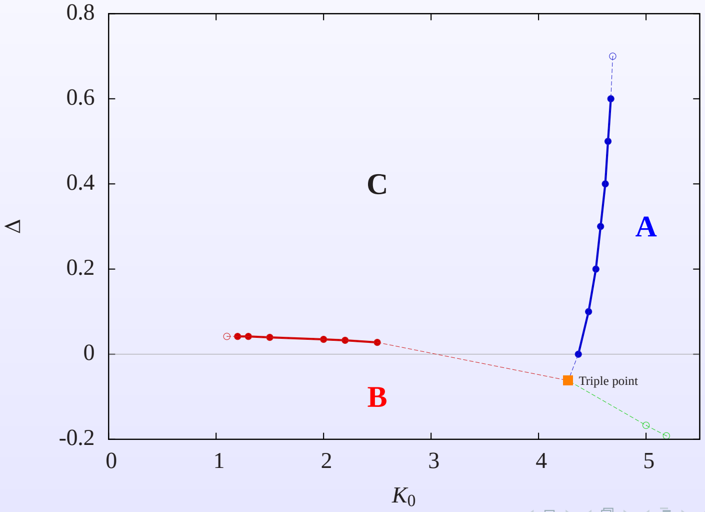
- The simplest observable giving information about the geometry, is the **spatial volume** n_i defined as a number of tetrahedra building a three-dimensional slice $i = 1 \ldots T$.
- Restricting our considerations to the spatial volume n_i we reduce the problem to one-dimensional quantum mechanics.

3D spatial slices with topology S^3



Phase diagram

$$S[\mathcal{T}] = -K_0 N_0 + K_4 N_4 + \Delta(N_{14} - 6N_0)$$



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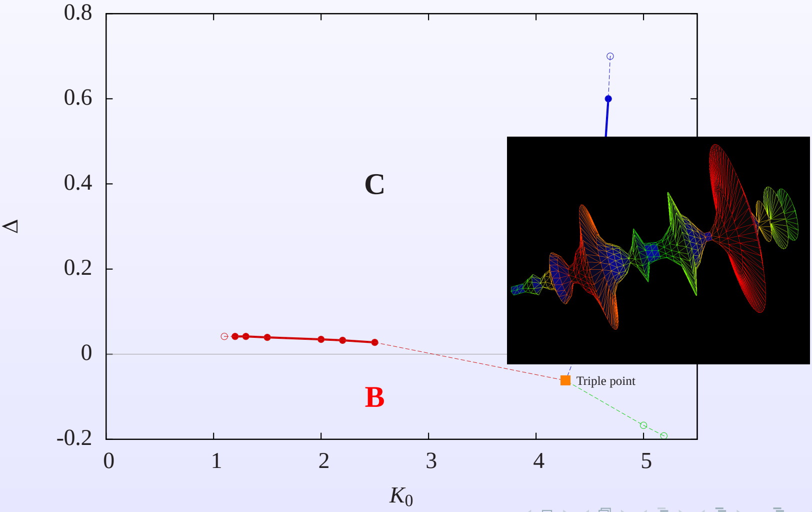

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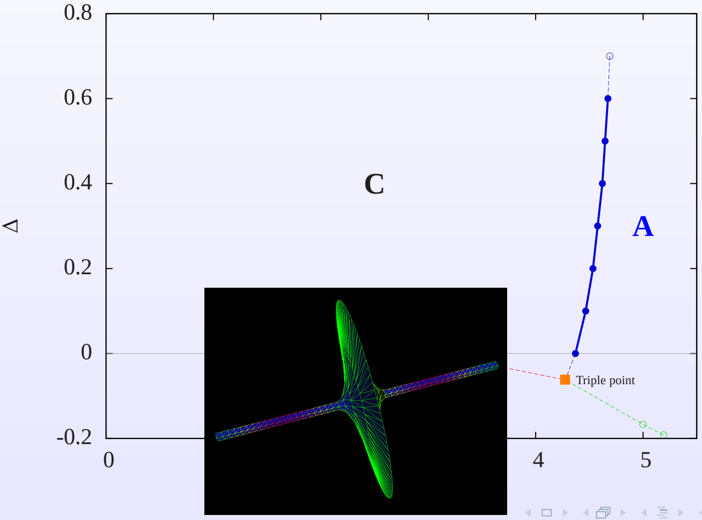

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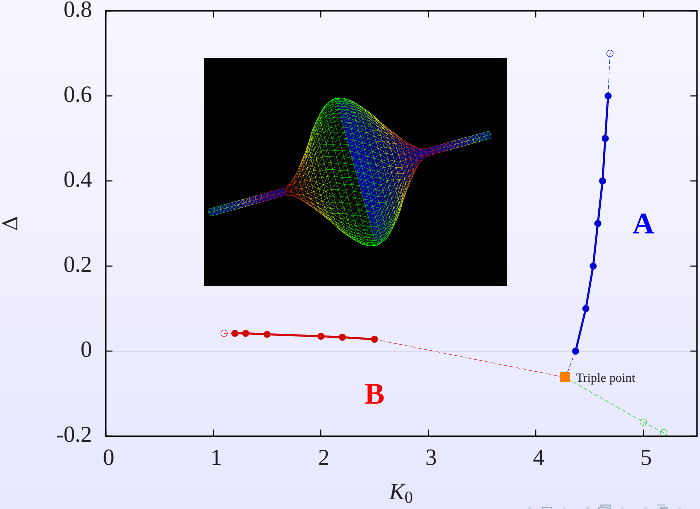
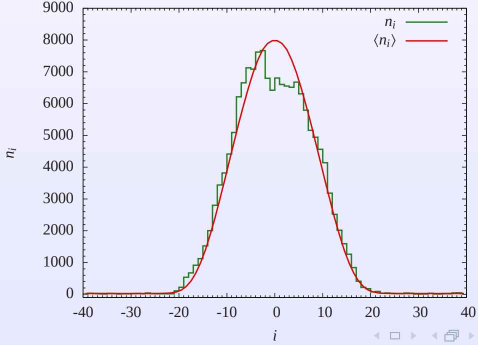


De Sitter space-time as background geometry

- In phase **C** the time translation symmetry is spontaneously broken and the distribution n_i is bell-shaped.
- The average volume $\langle n_i \rangle$ is with high accuracy given by formula

$$\langle n_i \rangle = H \cos^3\left(\frac{i}{W}\right)$$

- It describes Euclidean **de Sitter** space (S^4), a classical **vacuum solution**.



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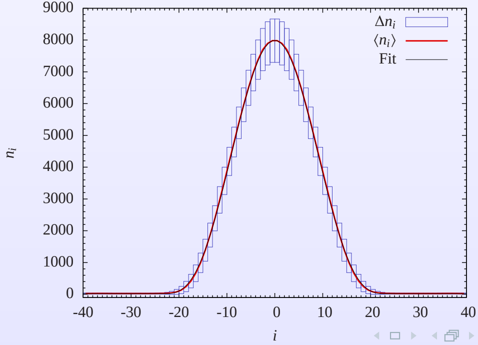
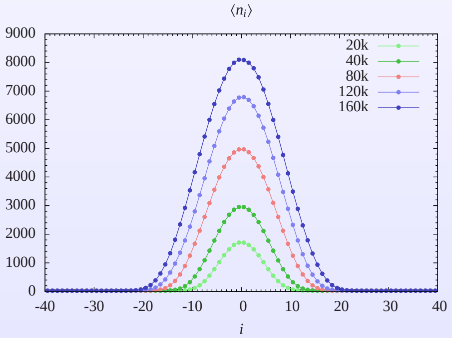


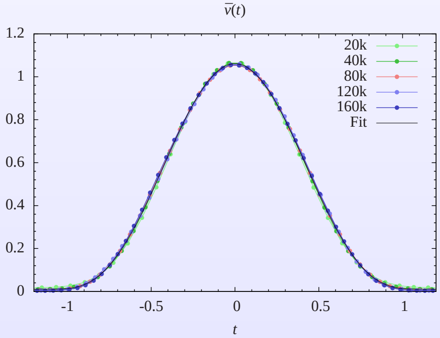
Hausdorff dimension

The time coordinate i and spatial volume $\langle n_i \rangle$ scale with total volume N_4 as a genuine four-dimensional *Universe*,

$$t = N_4^{-1/4}\, i,$$

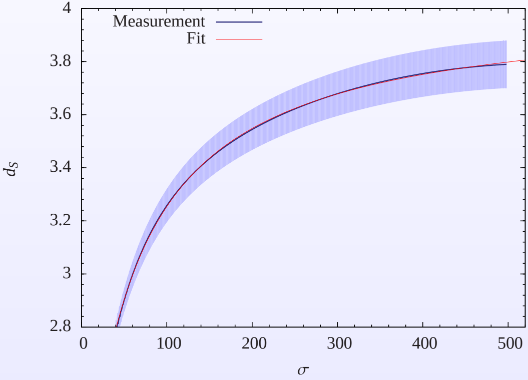
$$\bar{v}(t) = N_4^{-3/4} \langle n_i \rangle = \frac{3}{4\omega} \cos^3\left(\frac{t}{\omega}\right).$$





Spectral dimension

Simulations of the diffusion process allow to compute spectral dimension d_s.



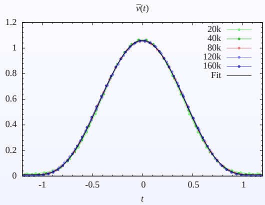
Extrapolation of the results gives short and long range behavior

$$d_s(\sigma \to 0) = 1.95 \pm 0.10, \quad d_s(\sigma \to \infty) = 4.02 \pm 0.10,$$

where σ is a fictitious diffusion time.

Minisuperspace model

$$\bar{v}(t) = \frac{3}{4\omega}\cos^3\left(\frac{t}{\omega}\right)$$



- Classical trajectory $\bar{v}(t)$ corresponds to Euclidean de Sitter space (S^4), a spatially homogeneous and isotropic vacuum solution.
- We „freeze" all degrees of freedom except volume and assume that metric on $S^3 \times S^1$ spacetime has form

$$\mathrm{d}s^2 = \mathrm{d}t^2 + a^2(t)\,\mathrm{d}\Omega_3^2, \quad v(t) = a^3(t)$$

- In this particular case, the Einstein-Hilbert action takes form

$$S = \frac{1}{G}\int \mathrm{d}t \int \mathrm{d}\Omega \sqrt{g}(R - 6\lambda)$$

with classical solution $\bar{v}(t)$.

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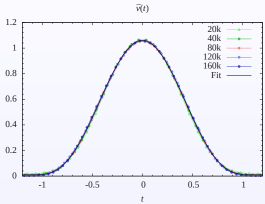

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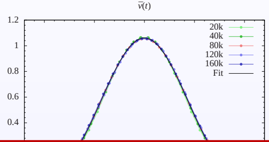

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Question?

How well does the minisuperspace model describe the quantum fluctuations computed from Monte Carlo simulations?

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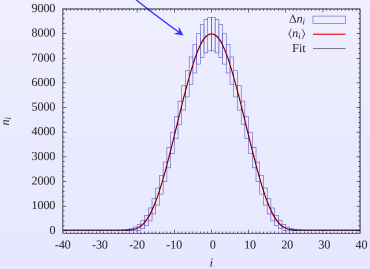
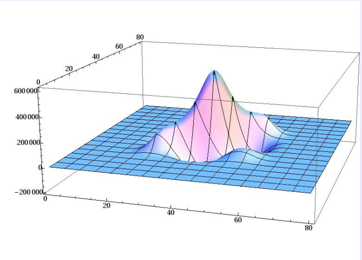
$$\mathbf{C}_{ij} \equiv \langle (n_i - \langle n_i \rangle)(n_j - \langle n_j \rangle) \rangle$$

- The propagator **C** appears in the **semiclassical** expansion of the **effective action** describing quantum fluctuations

$$S[n = \bar{n} + \eta] = S[\bar{n}] + \frac{1}{2} \sum_{i,j} \eta_i \, [\mathbf{C}^{-1}]_{ij} \, \eta_j + O(\eta^3), \quad [\mathbf{C}^{-1}]_{ij} = \left. \frac{\partial^2 S[n]}{\partial n_i \partial n_j} \right|_{n=\bar{n}}$$





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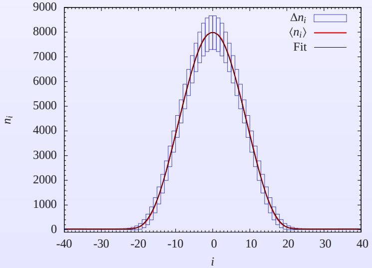

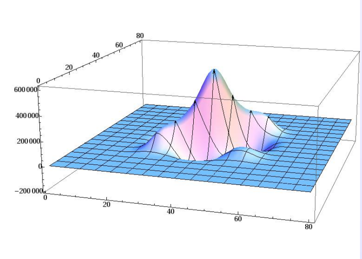

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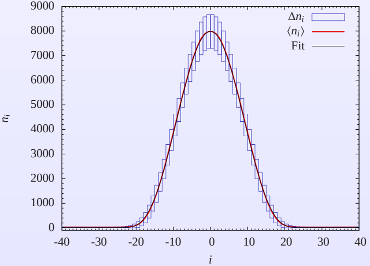
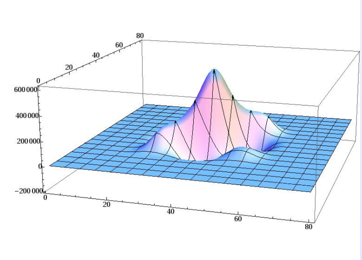




Effective action

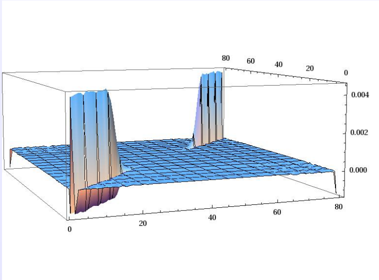
- Minisuperspace action $S[v] = \frac{1}{G}\int \frac{\dot{v}^2}{v} + v^{\frac{1}{3}} - \lambda v \mathrm{d}t$

$$\Downarrow$$

- Discretization $S[n] = \frac{1}{\Gamma}\sum_t \left(\frac{(n_{t+1}-n_t)^2}{n_{t+1}+n_t} + \mu n_t^{1/3} - \lambda n_t \right)$

- Inverse of propagator $\mathbf{C}$ (quadratic expansion)

$$\eta_i \boxed{[\mathbf{C}^{-1}]}_{ij}\, \eta_j = \frac{(\eta_{i+1}-\eta_i)^2}{k_i} - u_i \eta_i^2, \quad k_i \propto \bar{n}_i, \quad u_i \propto \bar{n}_i^{-5/3}$$



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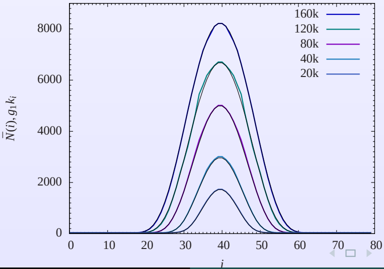

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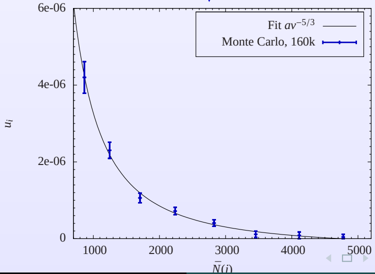
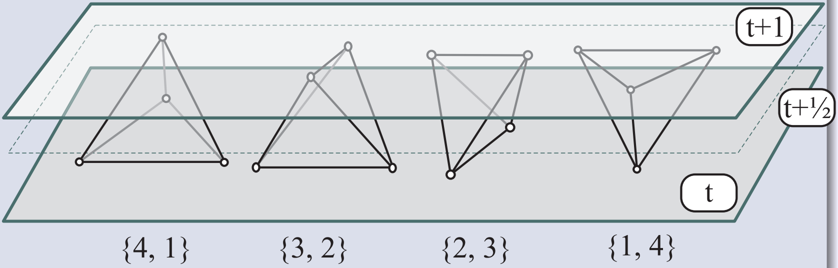


Refining the spatial slicing

- The slicing can be refined by taking into account the $\{3,2\}$ and $\{2,3\}$ simplices which are located between layers t and $t+1$ (simplices $\{4,1\}$ and $\{1,4\}$).
- Half integer times $t+1/2$ can be assign to $\{3,2\}$ simplices,

$$n_{t+\frac{1}{2}} \equiv \rho \cdot N_{32}(t), \quad n_t \equiv N_{41}(t).$$

Layers: $\{4,1\} - \{3,2\} - \{2,3\} - \{1,4\}$ simplices



Refining the spatial slicing

- The slicing can be refined by taking into account the $\{3,2\}$ and $\{2,3\}$ simplices which are located between layers t and $t+1$ (simplices $\{4,1\}$ and $\{1,4\}$).
- Half integer times $t+1/2$ can be assign to $\{3,2\}$ simplices,

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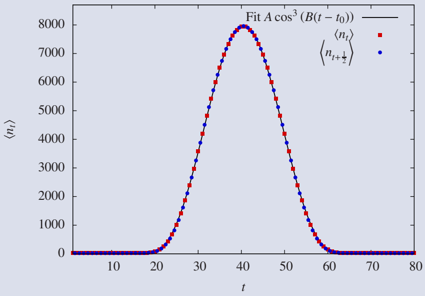
Volume profile for integer and half-integer t



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- Using the covariance matrix $\mathbf{C}$, we can determine the effective action

$$S_{eff} = \sum_{t\in\mathbb{Z}} \left(+ \frac{1}{\Gamma_1} \frac{\left(n_t - n_{t\pm\frac{1}{2}}\right)^2}{n_t + n_{t\pm\frac{1}{2}}} - \frac{1}{\Gamma_2} \frac{(n_t - n_{t+1})^2}{n_t + n_{t+1}} + V\left[n_t, n_{t+\frac{1}{2}}\right] \right)$$

- It occurs the $\{3,2\}$ simplices keep the slices together, while N_{41} repel each other. It is an entropic effect.
- **Integrating out** $N_{32}(t)$ recovers the original expression with

$$\frac{1}{\Gamma} = \frac{1}{2\Gamma_1} - \frac{1}{\Gamma_2}.$$

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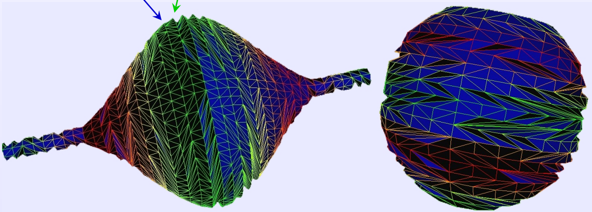
Transfer matrix

The model of Causal Dynamical Triangulations is completely determined by transfer matrix $\mathcal{M}$ labeled by 3D triangulations τ of spatial slices.

$$Z = \sum_{\mathcal{T}} e^{-S^R[\mathcal{T}]} \stackrel{!}{=} \mathrm{Tr}\mathcal{M}^T$$

$$P^{(T)}(\tau_1, \ldots, \tau_T) = \frac{1}{Z}\langle\tau_1|\mathcal{M}|\tau_2\rangle\langle\tau_2|\mathcal{M}|\tau_3\rangle\ldots\langle\tau_T|\mathcal{M}|\tau_1\rangle$$

Matrix element $\boxed{\langle\tau_1|}\mathcal{M}\boxed{|\tau_2\rangle}$ denotes the transition amplitude from state $|\tau_1\rangle$ to $|\tau_2\rangle$.



Effective transfer matrix

$$P^{(T)}(\tau_1, \ldots, \tau_T) \equiv \frac{1}{Z} \langle \tau_1 | \mathcal{M} | \tau_2 \rangle \langle \tau_2 | \mathcal{M} | \tau_3 \rangle \ldots \langle \tau_T | \mathcal{M} | \tau_1 \rangle$$

- The effective action obtained from the covariance matrix

$$S[n] = \frac{1}{\Gamma} \sum_t \left(\frac{(n_{t+1} - n_t)^2}{n_{t+1} + n_t} + \mu n_t^{1/3} - \lambda n_t \right)$$

  suggests existence of an effective transfer matrix labeled by the scale factor.

- Misleading to think of aggregated state $|n\rangle = \sum_{\tau \sim n} |\tau\rangle$
- Density operator $\rho(n) = |n\rangle\langle n| \equiv \sum_{\tau \sim n} |\tau\rangle\langle\tau|$

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Effective transfer matrix

$$P^{(T)}(n_1, \ldots, n_T) = \frac{1}{Z} \boxed{\langle n_1 | M | n_2 \rangle} \langle n_2 | M | n_3 \rangle \ldots \langle n_T | M | n_1 \rangle$$

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$$S[n] = \frac{1}{\Gamma} \sum_t \left(\frac{(n_{t+1} - n_t)^2}{n_{t+1} + n_t} + \mu n_t^{1/3} - \lambda n_t \right)$$

suggests existence of an effective transfer matrix labeled by the scale factor.

Effective transfer matrix $\langle n | M | m \rangle$ labeled by scale factor

$$\langle n | M | m \rangle = \mathcal{N} e^{-L_{eff}(n,m)}$$

$$S[n] = \sum_t L_{eff}(n_t, n_{t+1})$$

$$L_{eff}(n, m) = \frac{1}{\Gamma} \left[\frac{(n-m)^2}{n+m} + \mu \left(\frac{n+m}{2} \right)^{1/3} - \lambda \frac{n+m}{2} \right]$$

Assuming that

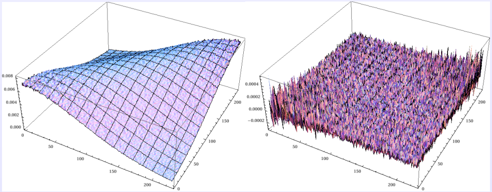
$$P^{(T)}(n_1, \ldots, n_T) = \frac{1}{Z}\langle n_1|M|n_2\rangle\langle n_2|M|n_3\rangle \ldots \langle n_T|M|n_1\rangle$$

we can measure elements of M

$$\langle n|M|m\rangle = \sqrt{P^{(2)}(n,m)} \text{ or } \langle n|M|m\rangle = \frac{P^{(3)}(n_1=n, n_2=m)}{\sqrt{P^{(4)}(n_1=n, n_3=m)}}$$

and check that it is consistent minisuperspace model

$$\langle n|M|m\rangle = \mathcal{N} e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m} + \mu\left(\frac{n+m}{2}\right)^{1/3} - \lambda\frac{n+m}{2}\right]}$$



Kinetic part

The effective action consists of kinetic and potential terms,

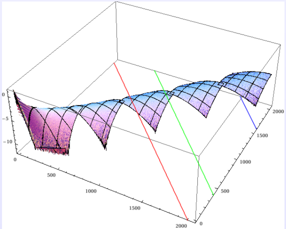
$$\langle n|M|m\rangle = \mathcal{N} e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m} + v\left(\frac{n+m}{2}\right)\right]}, \quad v(x) = \mu x^{1/3} - \lambda x + \delta x^{-\rho}$$

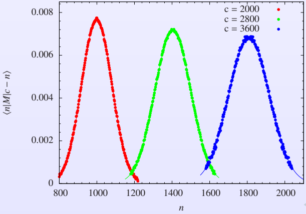
Gaussian
for $n + m = c$

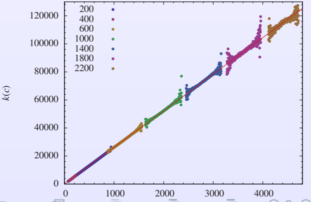
The kinetic term causes a Gaussian behavior for $n + m = c$

$$\langle n|M|c-n\rangle = \mathcal{N}(c) e^{-\frac{(2n-c)^2}{k(c)}}, \quad k(c) = \Gamma \cdot c$$

$\Gamma \approx 26.1$ is constant for all ranges of n.







Potential part

The effective action consists of kinetic and potential terms,

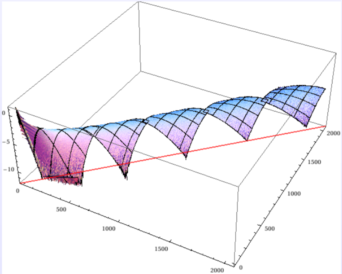
$$\langle n|M|m\rangle = \mathcal{N} e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m}+v\left(\frac{n+m}{2}\right)\right]}, \quad v(x) = \mu x^{1/3} - \lambda x + \delta x^{-\rho}$$

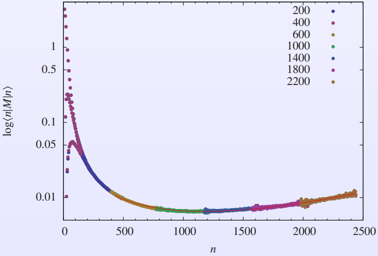
minisuperspace potential

possible curvature corrections

The potential part can be extracted from gathered data for $n = m$

$$\log\langle n|M|n\rangle = -\frac{1}{\Gamma}\left(\mu n^{1/3} - \lambda n + \delta x^{-\rho}\right) + \mathrm{const}$$





An effective model

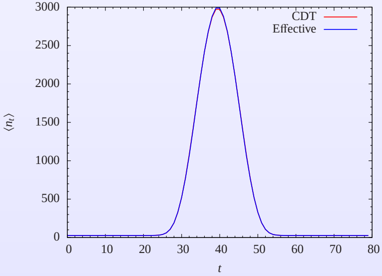
Let's consider a toy model in which states are given by volume profiles $\{n_t\}$ rather than triangulations,

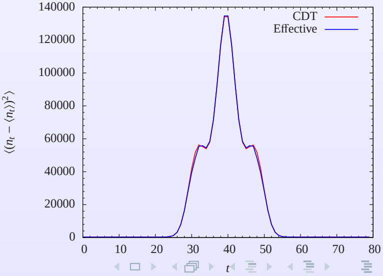
$$P^{(T)}(n_1, \dots, n_T) \propto \langle n_1|M|n_2\rangle\langle n_2|M|n_3\rangle \cdots \langle n_T|M|n_1\rangle,$$

where the transfer matrix elements are given by

$$\langle n|M|m\rangle = \mathcal{N} e^{-\frac{1}{\Gamma}\left[\frac{(n-m)^2}{n+m} + v\left(\frac{n+m}{2}\right)\right]}, \quad v(x) = \mu x^{1/3} - \lambda x + \delta x^{-\rho}.$$

It reproduces the full CDT results very accurately, supporting the transfer matrix approach.





Summary

1. The model of Causal Dynamical Triangulation is manifestly **diffeomorphism-invariant**. No coordinates are introduced and only geometric invariants, like length or angle, are involved.
2. Phase diagram consists of three phases. In phase **C** emerges a **four-dimensional** universe with well defined time and space extent.
3. The **background** geometry corresponds to the Euclidean **de Sitter** space, i.e. **classical solution** of the minisuperspace model.
4. The **effective action** obtained from both the correlation matrix and transfer matrix corresponds to the **minisuperspace** action and properly desribes the emergent background geometry and quantum fluctuations. In CDT no degrees of freedom are frozen.
5. The **semiclassical** limit is a truly **nonperturbative** limit. The contribution coming from the action is as important as the entropy of geometric configurations.

Thank You!

Based on:

J. Ambjørn, J. Gizbert-Studnicki, A. Görlich, J. Jurkiewicz, T. Trześniewski, *Nucl. Phys. B* **849**, *144 (2011)*.

J. Ambjørn, J. Gizbert-Studnicki, A. Görlich, J. Jurkiewicz, *JHEP* **09** *(2012) 017*