Gravity induced by NC spacetime

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Class. Quant. Gravity
31 (2014) 035020 (39pp)

Classical geometry ← Quantum geometry ← Quantum Gravity???

We are going to study this quantum spacetime

$$[r, t] = \imath\lambda_P r$$

- find a unique quantum metric (one scale parameter)
- a gravitational source so strong that even light cannot escape
- a Ricci curvature singularity at $r = 0$ even in the commutative limit $\lambda_P \to 0$
- a fully solved example of a 2D noncommutative Riemannian geometry; some purely noncommutative phenomena

I Background to the algebra

$m: \quad [x_i, t] = \imath \lambda_P x_i$

SM & H. Ruegg PLB 334 (1994)

Born reciprocity

SM Class. Quant. Gravity 5 (1988)

	Position	Momentum
Gravity	Curved	Noncommutative
Cogravity	Noncommutative	Curved
Quantum Gravity	Both	Both

SM Lect Notes Phys. 447 (1995)

SM Lect Notes Phys. 451 (2000)

Qua. Fou. Trans.

$\overline{U(m)} \longleftrightarrow C(M)$

Quantum spacetime hypothesis

Curved momentum space hypothesis

$C(M) \blacktriangleright\!\!\triangleleft U(so_{3,1})$ acting on $U(m)$ — semidual'n — $U(so_{3,1} \bowtie m)$ acting on $C(M)$

$C(SU_2) \rtimes\!\!\!\lhd U(su_2)$ acting on $U(su_2)$ $\longleftrightarrow$ $U(su_2 \oplus su_2)$ acting on $C(SU_2)$

bicrossproduct quantum group

factorising (quantum) group

- See this in 3D QG

SM & B. Schroers J. Phys A 42 (2009) 425402

II Background to the physics: wave operators

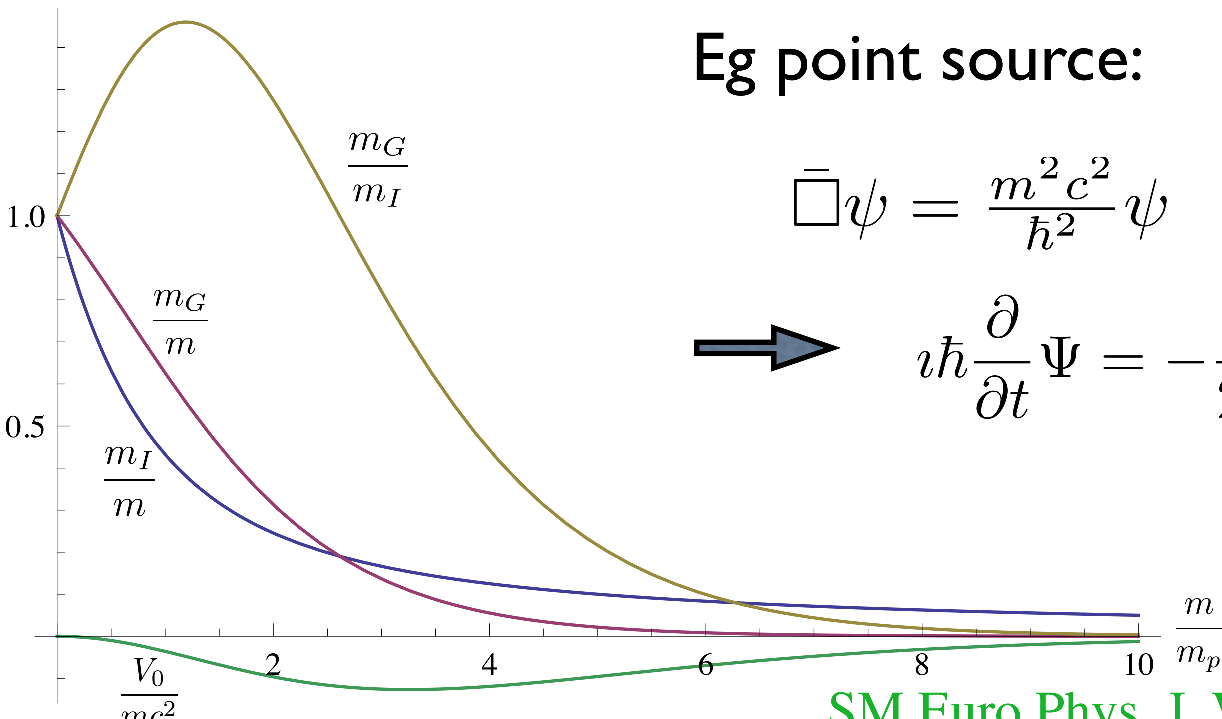
- $$[p^i, N_j] = -\frac{\imath}{2}\delta^i_j\left(\frac{1-e^{-2\lambda p^0}}{\lambda} + \lambda\vec{p}^2\right) + \imath\lambda p^i p_j,$$

$$||p||^2_\lambda = \vec{p}^2 e^{-\lambda p^0} - \frac{2}{\lambda^2}(\cosh(\lambda p^0) - 1)$$

Wave operator on nc plane waves $e^{i\vec{x}\cdot\vec{p}}e^{itp_0}$ ⟹ $|\frac{\partial p^0}{\partial p^i}| = e^{-\lambda p^0}$

G. Amelino-Camelia & S. M, Int. J. Mod. Phys. A 15 (2000)

- Freedom in extended differential structure = newtonian gravity



Eg point source:

$$\bar{\Box}\psi = \frac{m^2c^2}{\hbar^2}\psi \qquad \psi = \Psi(x,t)e^{-\imath\frac{mc^2}{\hbar}t}$$

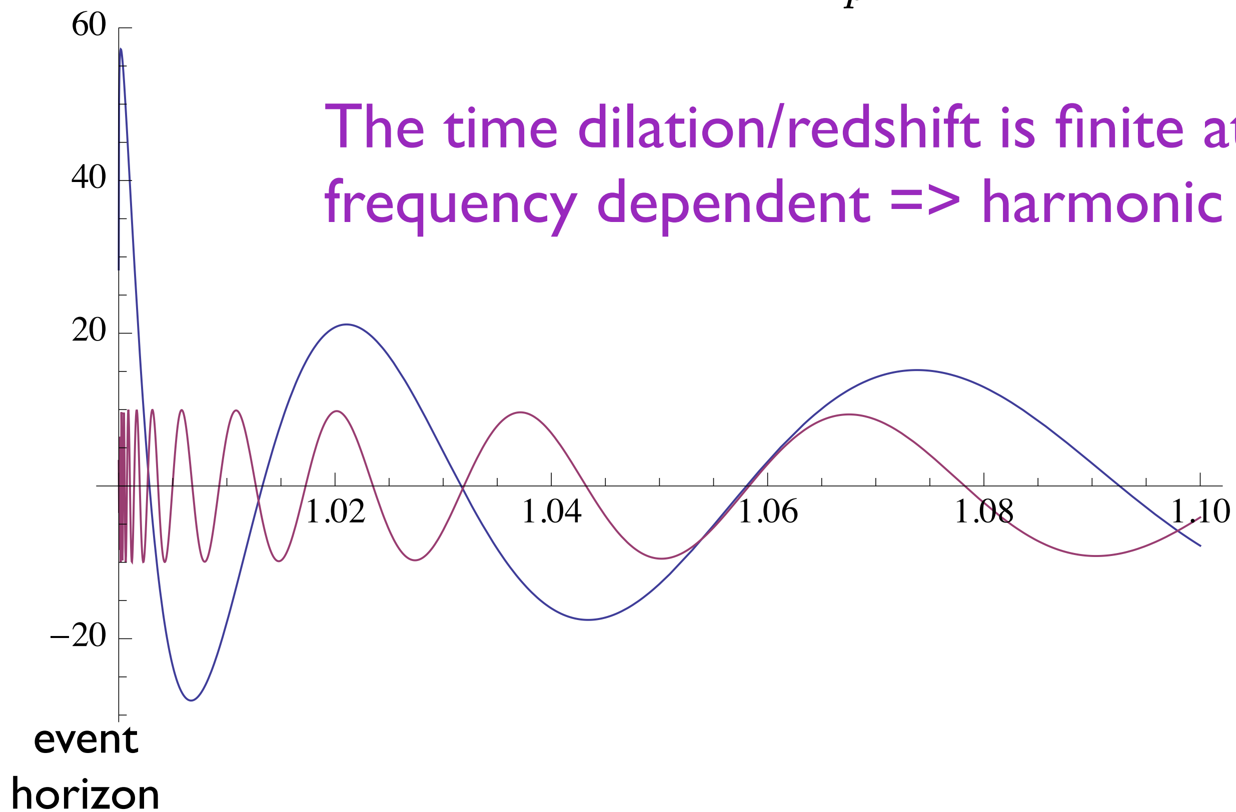
$$\Longrightarrow \quad \imath\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m_I}\bar{\Delta}^{flat}\Psi + (V_0 - \frac{GMm_G}{r})\Psi$$

SM Euro Phys. J. Web of Conferences, **70** (2014) 00082

- BH potential + minimal coupling => quantum Schw. black hole wave operator

$$\Rightarrow \quad \lim_{r\to\gamma} D(\omega, r) = \frac{\sinh(\omega\lambda_p)}{\lambda_p^2}$$

S.M Commun. Math. Phys. **310** (2012)



The time dilation/redshift is finite at the event horizon and frequency dependent => harmonic multiples destroyed by gravity

Thm $(M,\bar g)$ Riemannian, τ conformal killing, β a function => $\exists$ noncommutative $M\times\mathbb{R}$ relations: $[f,t]=\lambda\tau(f)$ with extended calc s.t $\Box$ quantises static spacetime $\beta^{-1}\mathrm{d}t\otimes\mathrm{d}t+\bar g$

III Quantum differentials on an algebra

- space of 1-forms, i.e. `differentials dx'

Ω^1 a((db)c)=(a(db))c `bimodule'

$\mathrm{d}: A \to \Omega^1$ d(ab)=(da)b+a(db) `Leibniz rule'

$\{\sum a db\} = \Omega^1$ `surjectivity'

- require this to extend to a DGA

$\Omega = T_A\Omega^1/\mathcal{I} = \oplus_n \Omega^n, \quad \mathrm{d}^2 = 0$ graded Leibniz rule

Prop.

bicovariant surjective

$\Omega^1(U(\mathfrak{g})) \quad \longleftrightarrow \quad \zeta \in Z^1(\mathfrak{g}, \Lambda^1)$

$\mathrm{d}x = 1 \otimes \zeta(x), \;\; \Omega^1 = U(\mathfrak{g}) \otimes \Lambda^1$

$\Rightarrow \;\; \Omega(U(\mathfrak{g}))$ SM+W.Tao

e.g. pre-Lie algebra

$\circ : \mathfrak{g} \otimes \mathfrak{g} \to \mathfrak{g} \quad [x, y] = x \circ y - y \circ x$

$(x \circ y) \circ z - (y \circ x) \circ z = x \circ (y \circ z) - y \circ (x \circ z)$

$\Lambda^1 \cong \mathfrak{g} \qquad x.\zeta(y) = \zeta(x \circ y)$

$\mathfrak{g}: \quad [r,t] = \lambda r$

canonical calculus (unique *-preserving 2D)

$A = U(\mathfrak{g})$

$\Omega^1 = A\mathrm{d}r \oplus A\mathrm{d}t$

$\Lambda^1 = \mathfrak{g} \qquad \zeta \text{ exact}$

$[r, \mathrm{d}t] = \lambda \mathrm{d}r, \quad [t, \mathrm{d}t] = \lambda \mathrm{d}t$

$$\Longrightarrow \qquad \mathrm{d}f(r,t) = \frac{\partial f}{\partial r}\mathrm{d}r + (\partial_0 f)\mathrm{d}t, \quad \partial_0 f = \frac{f(r,t) - f(r,t-\lambda)}{\lambda}$$

$$\Box = \partial_0^2 - (\frac{\partial}{\partial r})^2$$

Variable speed of light prediction

Note

$$[f(r), t] = \lambda r \frac{\partial f}{\partial r} \qquad [r, f(t)] = \lambda r \partial_0 f \quad \xrightarrow{\lambda \neq 0} \quad Z(A) = \mathbb{C}1$$

$$[f, \mathrm{d}r] = 0, \quad [f, \mathrm{d}t] = \lambda \mathrm{d}f \quad \Longrightarrow \quad \mathrm{d}r, \quad v = r\mathrm{d}t - t\mathrm{d}r$$

central

IV Quantum metric tensor

$$g \in \Omega^1 \underset{A}{\otimes} \Omega^1 \qquad \wedge(g) = 0$$ \`quantum symmetric'

invertible in the sense exists inverse: $(\ , \ ) : \Omega^1 \underset{A}{\otimes} \Omega^1 \to A$

$$((\ ,\ ) \otimes \mathrm{id})(\omega \otimes g) = \omega = (\mathrm{id} \otimes (\ ,\ ))(g \otimes \omega), \quad \forall \omega \in \Omega^1$$

$$a(\omega, \eta) = (a\omega, \eta), \quad (\omega, \eta)a = (\omega, \eta a)$$ \`bimodule map (tensorial)'

need this to be able to contract/ \`raise/lower' via metric, eg to have well defined contraction:

$$(\ ,\ ) \otimes \mathrm{id} : \ \Omega^1 \underset{A}{\otimes} \Omega^1 \underset{A}{\otimes} \Omega^1 \to \Omega^1$$ " $T_{\mu\nu\rho} \mapsto g^{\mu\nu} T_{\mu\nu\rho}$ "

but

$$(\omega, g^1) g^2 = \omega \qquad g = g^1 \underset{A}{\otimes} g^2$$

$$\Longrightarrow (\omega, g^1) g^2 a = \omega a = (\omega a, g^1) g^2 = (\omega, a g^1) g^2$$

$$\Longrightarrow \qquad ag = ga, \quad \forall a \in A$$ need metric to be central

Lemma: In our DGA a quantum metric has the form

$$g = (t^2 + 2\beta t + \lambda t + \alpha)\mathrm{d}r \otimes \mathrm{d}r - r(t+\beta)(\mathrm{d}r \otimes \mathrm{d}t + \mathrm{d}t \otimes \mathrm{d}r) + r^2 \mathrm{d}t \otimes \mathrm{d}t$$

(proof depends on $\wedge g = 0, \quad [g, r] = [g, t] = 0$)

* Algebras

work over $\mathbb{C}$ but specify real differential geometry via

- $* : A \to A$ antilinear involution `*-algebra'
- extends to graded-anti-algebra hom on $\Omega(A), \ [*, \mathrm{d}] = 0$
- metric hermitian in sense $(* \otimes *)(g) = \mathrm{flip}(g)$

Propn.: In our DGA $r^* = r, \ t^* = t, \ \lambda^* = -\lambda$. A quantum metric has the unique form

$$g = \mathrm{d}r \otimes \mathrm{d}r + b(v^* \otimes v + \lambda(\mathrm{d}r \otimes v - v^* \otimes \mathrm{d}r))$$

$v = r\mathrm{d}t - t\mathrm{d}r, \quad v^* = (\mathrm{d}t)r - t\mathrm{d}r \qquad b \in \mathbb{R} \quad b \neq 0$

V Classical limit $\lambda \to 0$

$$g = \mathrm{d}r \otimes \mathrm{d}r + bv \otimes v = (1+bt^2)\mathrm{d}r^2 + br^2\mathrm{d}t^2 - 2brt\mathrm{d}r\,\mathrm{d}t$$

$$\Longrightarrow \quad \nabla \mathrm{d}r = \frac{2b}{r} v \otimes v, \quad \nabla v = -\frac{2}{r} v \otimes \mathrm{d}r \quad \leftrightarrow \quad \Gamma^a_{bc}$$ Christoffel symbols

- $$\mathrm{Ricci} = \frac{g}{r^2}$$ curvature singularity at r=0!

- Geodesic equations

$$\ddot{x}^a = -\Gamma^a_{bc}\,\dot{x}^b\,\dot{x}^c \Longrightarrow M = r^2(r\dot{t} - t\dot{r})$$ is a constant of motion

(a) $b < 0$ case:

timelike geodesics, with slope c at $r = 0$ starting at $r = -D, \quad D = (-bM^2)^{\frac{1}{4}}$

$$\tau(r) = D\gamma + DE(\frac{r}{D}) \quad \gamma = E(1) = \sqrt{\pi}\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} \approx 0.59907 \quad E(x) = \int_0^x \frac{u^2}{\sqrt{1-u^4}}\mathrm{d}u$$

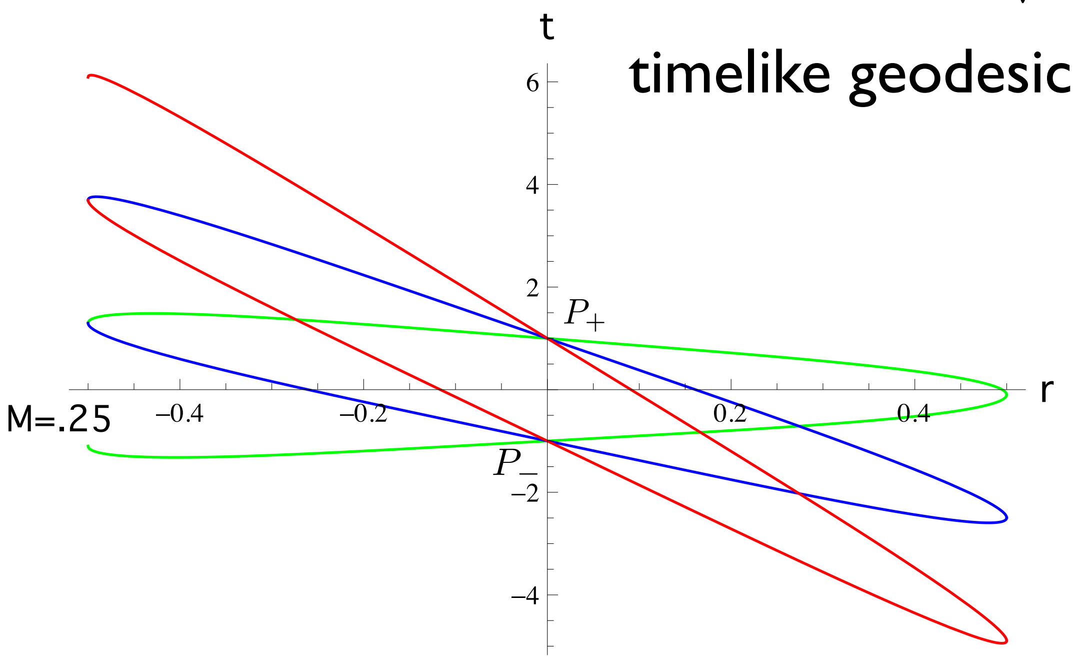
$$t_n(r) = rc - 2n\frac{M\gamma}{D^3}r - (-1)^n\frac{M}{D^2}\left(\sqrt{1-(\frac{r}{D})^4} + \frac{r}{D}E(\frac{r}{D})\right), \quad \forall |r| \le D, \quad n \in \mathbb{Z},$$

for the segment where

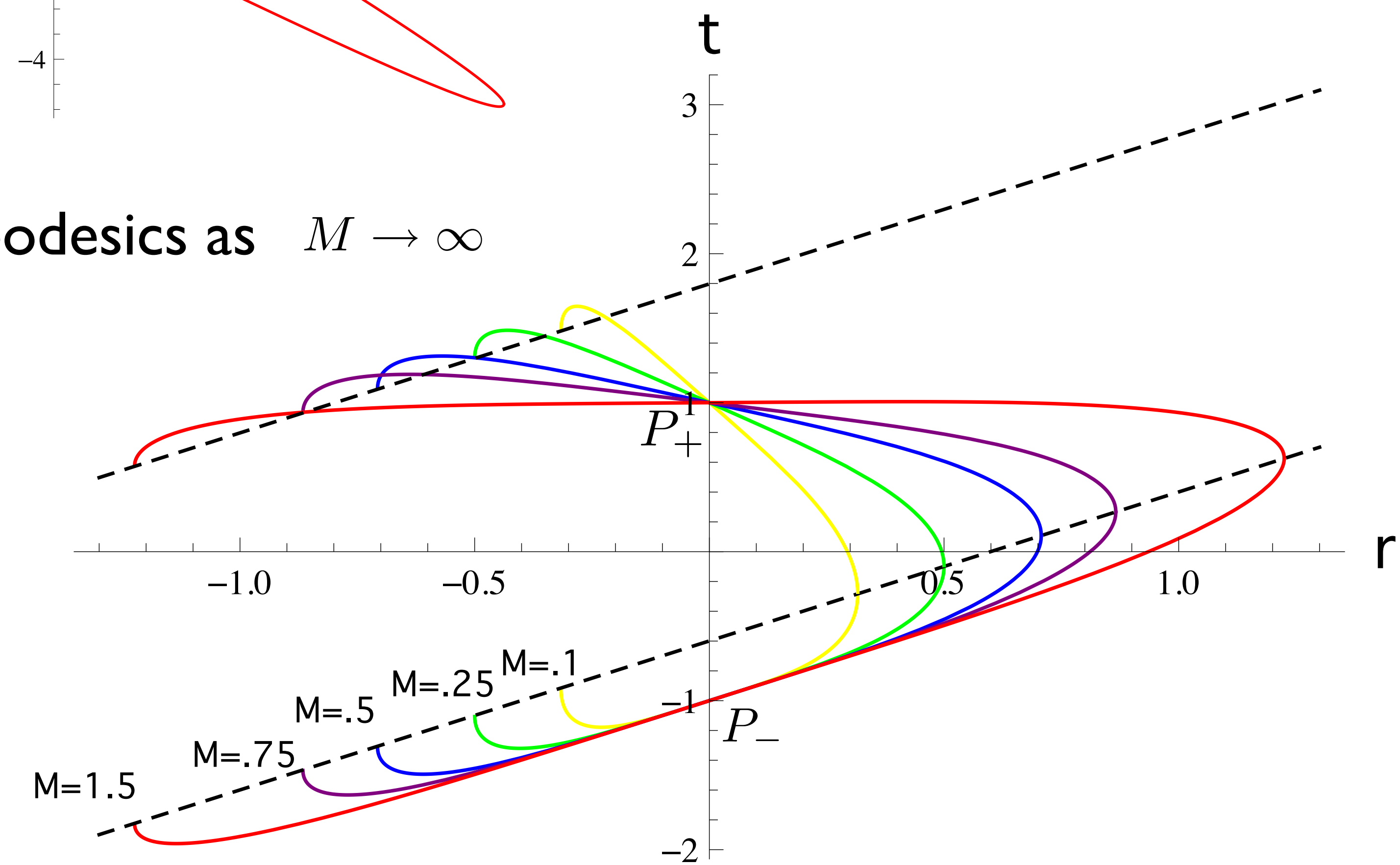
$$n2D\gamma \le \tau \le (n+1)2D\gamma.$$

All geodesics pass through $P_{\pm} = (0, \pm\frac{1}{\sqrt{-b}})$

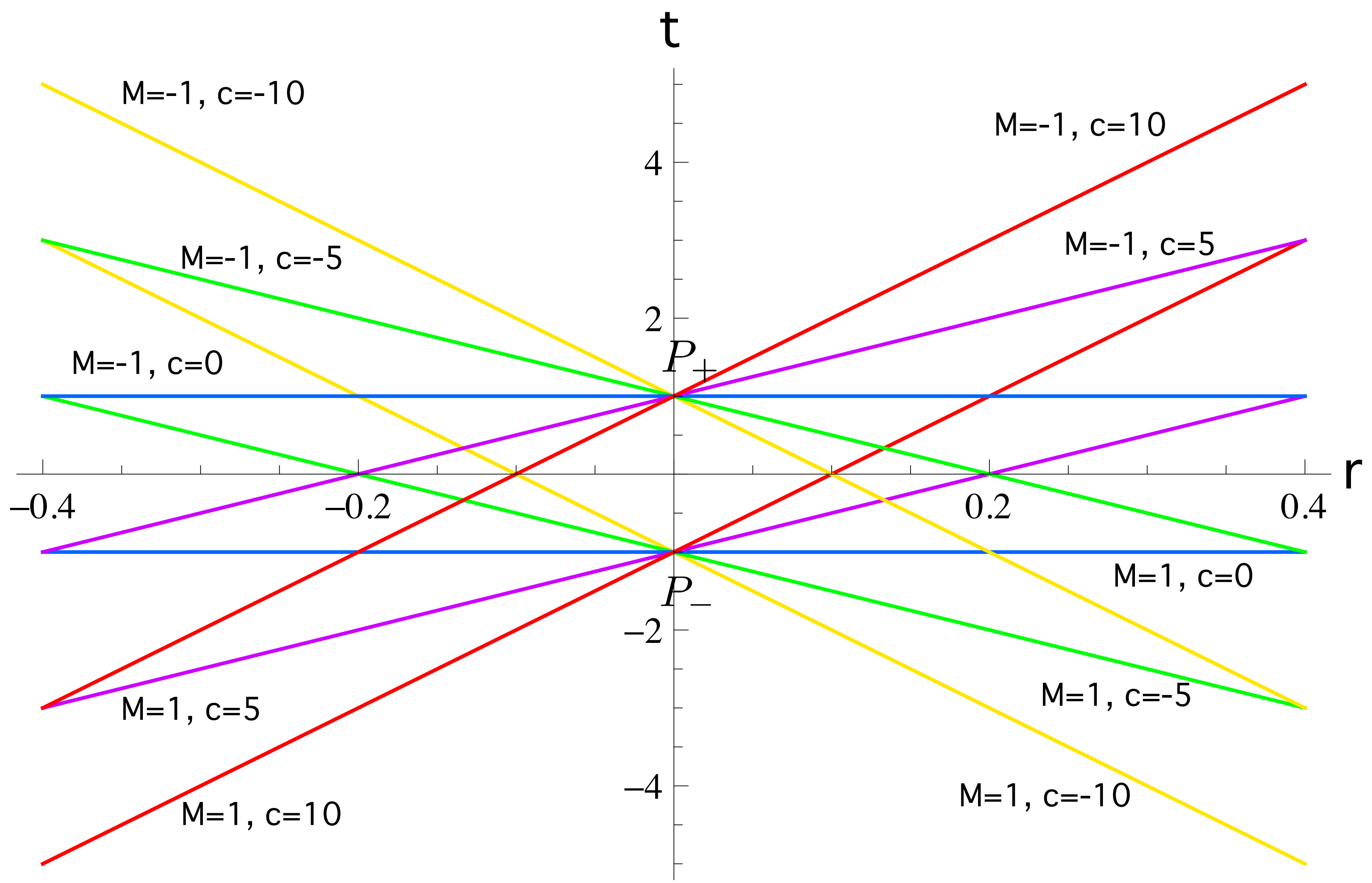
timelike geodesics bounce between $r = \pm(-bM^2)^{\frac{1}{4}}$



and become null geodesics as $M \to \infty$



null geodesics slope c: $t = rc - \frac{M}{\sqrt{-bM^2}}$



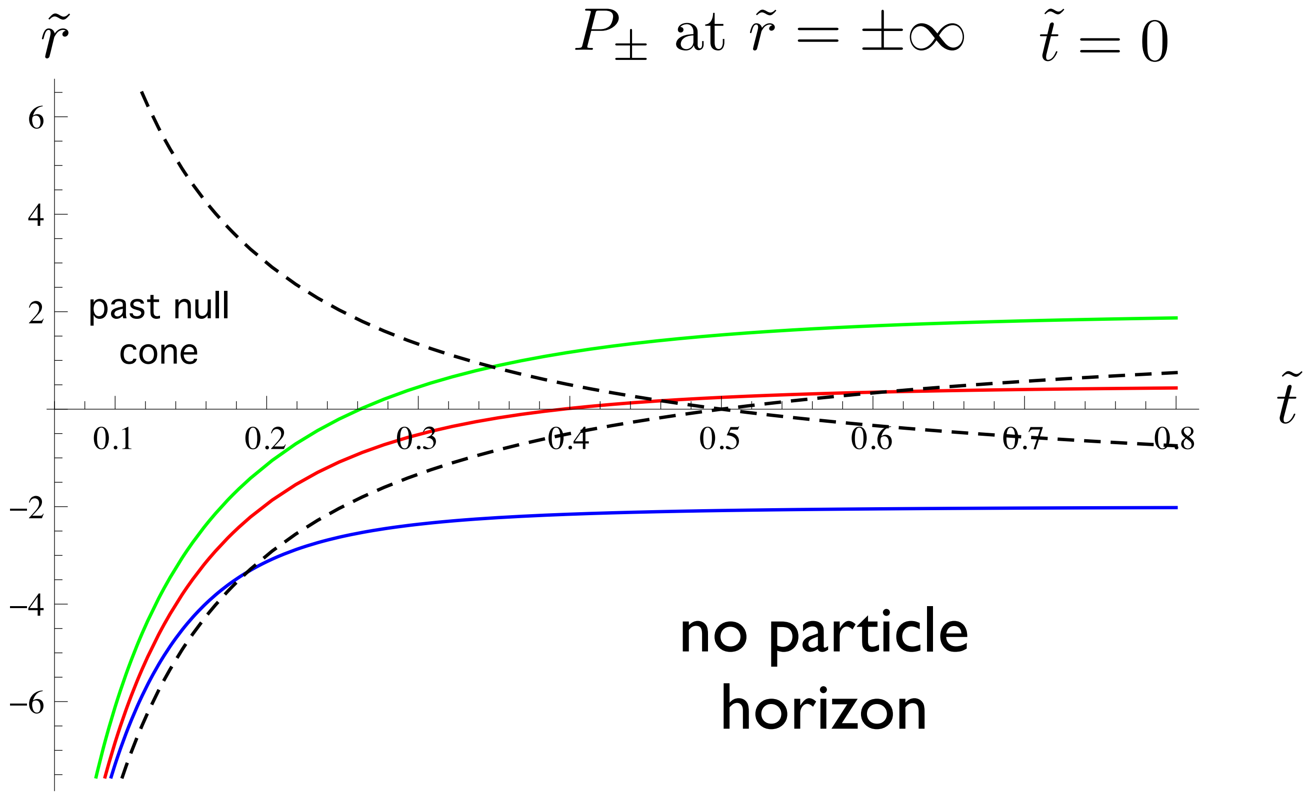
So even an outgoing photon bounces back in at $r = \infty$ `black hole'

(b) $b > 0$ case: use new FRW-like coordinates $\tilde{t} = r, \quad \tilde{r} = \frac{t}{r}$

$$g = -\mathrm{d}\tilde{t}^2 + R(\tilde{t})^2 \mathrm{d}\tilde{r}^2, \quad R(\tilde{t}) = \sqrt{b}\tilde{t}^2$$

All geodesics start/end on

$P_{\pm}$ at $\tilde{r} = \pm\infty$ $\quad \tilde{t} = 0$



no particle horizon

ricci singularity

$\tilde{t} = 0$

3+1 case

$$[x_i, t] = \lambda x_i, \quad [x_i, x_j] = 0 \qquad [f, \mathrm{d}t] = \lambda \mathrm{d}f$$

Thm: no central metrics exist at all on this differential algebra

But can find central w.r.t. functions in $r = |\vec{x}|, t$ => 2-parameter family

$$g = r^2 \mathrm{d}\Omega + a\mathrm{d}r \otimes \mathrm{d}r + b\left(v^* \otimes v + \lambda(\mathrm{d}r \otimes v - v^* \otimes \mathrm{d}r)\right) \qquad a, b \in \mathbb{R}$$

=> $$G_{ij} = R_{ij} - \tfrac{1}{2} S\, g_{ij} = \begin{pmatrix} \left(\frac{1}{a} - 1\right) b & \frac{(a-1)bt}{ar} & 0 & 0 \\ \frac{(a-1)bt}{ar} & \frac{-a^2 - bt^2 a + 5a + bt^2}{ar^2} & 0 & 0 \\ 0 & 0 & \frac{4}{a} & 0 \\ 0 & 0 & 0 & \frac{4\sin^2(p)}{a} \end{pmatrix}$$

$$G^{ij} = \frac{8\pi G}{c^4} T^{ij}$$

$$T^{ij} = p\, g^{ij} + (p + \rho)\, u^i u^j$$

$$g_{ij} u^i u^j = -1$$

} =>

a=1, b<0: $p = 1/(2\pi G r^2), \quad \rho = 0$

a=-3, b>0: $p = -1/(6\pi G r^2), \quad \rho = 1/(3\pi G r^2)$

(quintessence $\frac{p}{\rho} = -\frac{1}{2}$)

=> null geodesics spiral out from $r = 0$ to $r = 0, \infty$ respectively.

=> metric conformal scaling of flat $\mathbb{R}^{1,3}$ metric in new coordinates.

VI Quantum Riemannian Geometry

bimodule connection $\nabla : \Omega^1 \to \Omega^1 \underset{A}{\otimes} \Omega^1$ $\quad\sigma : \Omega^1 \underset{A}{\otimes} \Omega^1 \to \Omega^1 \underset{A}{\otimes} \Omega^1$

(Michor, Dubois-Violette)

$$\nabla(f\omega) = \mathrm{d}f \otimes \omega + f\nabla\omega \qquad \nabla(\omega f) = \sigma(\omega \otimes \mathrm{d}f) + (\nabla\omega)f$$

action on 2-tensor

$$\omega \otimes \eta \in \Omega^1 \underset{A}{\otimes} \Omega^1 \qquad \nabla(\omega \otimes \eta) = \nabla\omega \otimes \eta + (\sigma \otimes \mathrm{id})(\omega \otimes \nabla\eta)$$

metric $\nabla g = 0$ `metric compatible' now makes sense

torsion $T_\nabla : \Omega^1 \to \Omega^2$ $\quad T_\nabla = \wedge\nabla - \mathrm{d}$ torsion free now makes sense

curvature $R_\nabla : \Omega^1 \to \Omega^2 \underset{A}{\otimes} \Omega^1$

$$R_\nabla = (\mathrm{d} \underset{A}{\otimes} \mathrm{id} - (\wedge \underset{A}{\otimes} \mathrm{id})(\mathrm{id} \underset{A}{\otimes} \nabla))\nabla$$

reality $\nabla(\xi) = \sigma(\zeta^* \otimes \eta^*), \quad \forall \eta \otimes \zeta = \nabla(\xi^*)$ on complex *-algebra

Method to find ∇ metric compatible

$$\nabla = \nabla_0 + \lambda\,\nabla_1 + O(\lambda^2) \qquad \nabla_0(\mathrm{d}r) = \frac{2\,b}{r}\,v\otimes v, \quad \nabla_0(v) = -\,\frac{2}{r}\,v\otimes \mathrm{d}r$$

$$\sigma(\omega\otimes \mathrm{d}a) \;=\; \mathrm{d}a\otimes\omega + [a,\nabla(\omega)] + \nabla([\omega,a]) \qquad \forall a\in A,\;\; \omega\in\Omega^1$$

to $O(\lambda)$ it is enough to calculate this using ∇_0

$$\Longrightarrow\; \sigma(\mathrm{d}r\otimes\mathrm{d}t) = \mathrm{d}t\otimes\mathrm{d}r + [t,\nabla_0(\mathrm{d}r)] = \mathrm{d}t\otimes\mathrm{d}r + \frac{2\,b\,\lambda}{r}\,v\otimes v\;,$$

$$\sigma(v\otimes\mathrm{d}t) = \mathrm{d}t\otimes v + [t,\nabla_0(v)] = \mathrm{d}t\otimes v - \frac{2\,\lambda}{r}\,v\otimes\mathrm{d}r\;.$$

$$\Longrightarrow\; \sigma(v\otimes v) = v\otimes v - 2\,\lambda\,v\otimes\mathrm{d}r\;, \qquad \sigma(\mathrm{d}r\otimes v) = v\otimes\mathrm{d}r + 2\,b\,\lambda\,v\otimes v$$

$$\sigma(v\otimes\mathrm{d}r) = \mathrm{d}r\otimes v, \qquad \sigma(\mathrm{d}r\otimes\mathrm{d}r) = \mathrm{d}r\otimes\mathrm{d}r\;.$$

Combine this with the reality constraint:

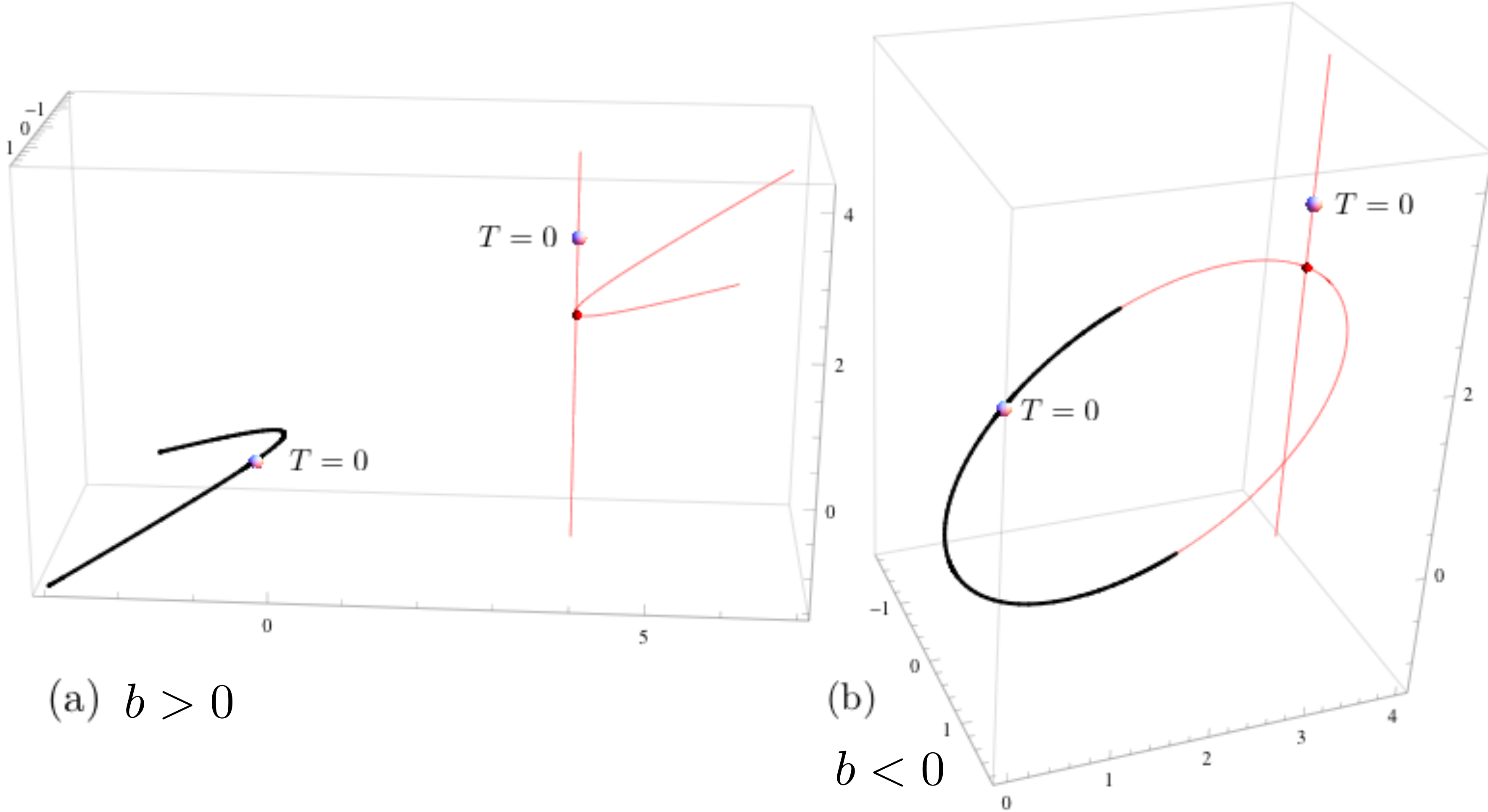
set $\eta_0\otimes\zeta_0 = \nabla_0(\xi^*)$ and $\eta_1\otimes\zeta_1 = \nabla_1(\xi^*)$.

$$\nabla_0(\xi) + \lambda\,\nabla_1(\xi) = \sigma(\zeta_0^*\otimes\eta_0^*) - \lambda\,\eta_1^*\otimes\zeta_1^* \qquad\qquad \xi^* \;=\; \xi \text{ to } O(\lambda^0)$$

$$\Longrightarrow\; \lambda\,(\eta_1\otimes\zeta_1 + \eta_1^*\otimes\zeta_1^*) = \sigma(\zeta_0^*\otimes\eta_0^*) - \nabla_0(\xi)$$ ... so can solve this iteratively

More generally, quadratic system for reality. Then impose metric compatibility

Propn. Moduli of real metric-compatible ∇ form a line + conic



- black parts have classical limit as $\lambda \to 0$
- red parts blow up as $\lambda \to 0$ so not visible classically
- in each case a unique `Levi-Civita point' where torsion T=0

- Unique Levi-Civita soln with classical limit:

$$\nabla \mathrm{d}r = \frac{1}{r}\left(v - \frac{\lambda \mathrm{d}r}{2}\right)\otimes\left((\frac{8b}{4+7b\lambda^2})v - (\frac{12b\lambda}{4+7b\lambda^2})\mathrm{d}r \right)$$

$$\mathrm{Ricci} \;=\left(\frac{4+7b\lambda^2}{4-9b\lambda^2}\right)\frac{g}{r^2}$$

$$\mathrm{Ricci} - \frac{1}{(\;,\;)(g)}S = 0, \quad S = (\;,\;)\mathrm{Ricci} \qquad (\;,\;)(g) = \frac{2+b\lambda^2}{1+b\lambda^2}$$

quantum dimension

⇒ `quantum Einstein tensor'=0

⇒ `usual Einstein' $\mathrm{Ricci} - \frac{1}{2}S = b\lambda^2\frac{g}{2r^2} + O(\lambda^3)$

Aside: $\mathrm{Ricci} = ((\;,\;)\otimes \mathrm{id})(\mathrm{id}\otimes i\otimes \mathrm{id})((\mathrm{id}\otimes R)(g)$

$i:\Omega^2\to\Omega^1\otimes_A\Omega^1$ uniquely determined by $\wedge i = \mathrm{id}$ and requirement that Ricci has same symmetry and reality as metric

- Unique Levi-Civita soln without classical limit:

$$\nabla \mathrm{d}r = \frac{bv}{r} \otimes \left((\frac{1}{1+b\lambda^2})v - (\frac{2}{\lambda})\mathrm{d}r \right) + (\frac{2+b\lambda^2}{r(1+b\lambda^2)})\mathrm{d}r \otimes \left(-(\frac{1}{\lambda})v + (\frac{3}{2})\mathrm{d}r \right)$$

blows up as $\lambda \to 0$ and its

$$\mathrm{Ricci} = -3\frac{(4+3b\lambda^2)(-2+b\lambda^2)}{20r^2\lambda^2\,(1+b\lambda^2)}(v^* \otimes v + \lambda(\mathrm{d}r \otimes v - v^* \otimes \mathrm{d}r)$$
$$+\frac{\left(1+b\lambda^2\right)\left(14+3b\lambda^2\right)}{b\left(-2+b\lambda^2\right)}\mathrm{d}r \otimes \mathrm{d}r)$$

nothing like the metric

It all works in our 2D example of `quantum' Riemannian geometry, but the metric can't be flat and there is a second non-deformation solution for the Levi-Civita connection

VII Semiquantisation of $C^\infty(M)$ w/ Beggs arXiv:1403.4231

$$a.b - b.a = \lambda\{a,b\} + O(\lambda^2)$$

ω^{ij} Poisson tensor

$\hat{a} = \{a, \ \}$ Hamilt. vec field

$$a.\mathrm{d}b - (\mathrm{d}b).a = \lambda\nabla_{\hat{a}}\mathrm{d}b + O(\lambda^2)$$

∇_i connection with torsion T

Thm (A): If $\omega^{ij}_{;m} + \omega^{ik}T^j_{km} - \omega^{jk}T^i_{km} = 0$

$\exists$ noncomm. DGA $\Omega(A)$ to O(λ), same d

Propn: The Poisson-compat, connection itself gets quantised to a bimodule connection $\nabla_Q : \Omega^1 \to \Omega^1 \otimes_1 \Omega^1$, $\sigma_Q : \Omega^1 \otimes_1 \Omega^1 \to \Omega^1 \otimes_1 \Omega^1$

$$\nabla_Q \mathrm{d}x^i = -\left(\Gamma^i_{mn} + \frac{\lambda}{2}\omega^{sj}(\Gamma^i_{mk,s}\Gamma^k_{jn} - \Gamma^i_{kt}\Gamma^k_{sm}\Gamma^t_{jn} - \Gamma^i_{jk}R^k{}_{nms})\right)\mathrm{d}x^m \otimes_1 \mathrm{d}x^n$$

and has a quantum torsion

$$T_{\nabla_Q}\xi = \frac{1}{2}(\xi_i T^i_{nm} + \tfrac{\lambda}{2}\,(\partial_j \lrcorner \nabla_i \xi)\,\omega^{is}\,T^j_{nm;s})\,\mathrm{d}x^m \wedge_1 \mathrm{d}x^n$$

Thm (B): $\exists$ monoidal functor to order λ

(Q,q): Bundles w. Conn. $\longrightarrow$ A-Bimod w. Bimod Conn.

Thm (C): suppose (ω, ∇) Poisson compat and metric g, $\nabla g = 0$. Let

$$\mathcal{R} = g_{ij}\omega^{is}(T^j_{nm;s} - 2R^j{}_{nms})\mathrm{d}x^m \wedge \mathrm{d}x^n$$ `generalised Ricci form'

$$g_1 := q^{-1}(g - \frac{\lambda}{4} g_{ij}\omega^{is}(T^j_{nm;s} - R^j{}_{nms} + R^j{}_{mns})\mathrm{d}x^m \otimes_0 \mathrm{d}x^n)$$ `quant metric'

(1) g_1 is quantum symmetric and central

(2) $\nabla_Q(g_1) = 0$ <=> $\nabla\mathcal{R} = 0$

Cor: If $\nabla = \widehat{\nabla}$ (the classical Levi-Civita) then ∇_Q is the quantum Levi-Civita iff $\mathcal{R} = -\frac{1}{2} g_{ij}\omega^{is}R^j{}_{nms}\mathrm{d}x^m \wedge \mathrm{d}x^n \in \Omega^2(M)$ is cov. constant.

Works any Kahler-Einstein manifold! eg $\mathbb{C}P^n$

Thm (D): $\exists!$ `best possible' quantum Levi-Civita ∇_1
in the sense torsion free and sym($\nabla_1 g_1$)=0

Full $\nabla_1 g_1 = 0$ iff

$$\widehat{\nabla}\mathcal{R} + \omega^{ij}\, g_{rs}\, S^s_{jn}(R^r{}_{mki} + S^r_{km;i})\, \mathrm{d}x^k \otimes \mathrm{d}x^m \wedge \mathrm{d}x^n = 0$$

In this case ∇_1 is also *-preserving. $\widehat{\nabla} = \nabla + S$

- For given ω there may not exist zero curv ∇
=> nonassociativity at O(λ^2) in DGA

- Given (ω, ∇) there may not exist g such that
$\nabla g = 0$ => quantum metric not central

- Given (ω, ∇, g) we have found an obstruction to construction of full quantum Levi-Civita

Works Schw BH: unique 4-funl param rotl invariant (ω,∇) but they all have curvature and levi-civita obstruction